8/29 p1

Suppose $\int_0^1 \rho(x)\,dx = 2$

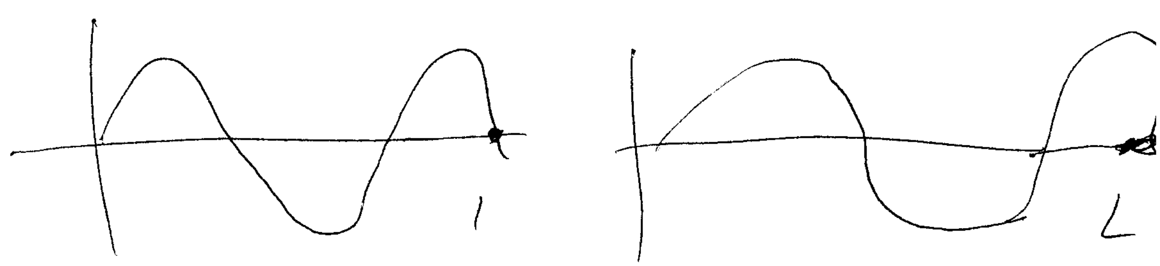
Find $\int_0^L \rho\left(\frac{x}{L}\right) dx$

$u = \frac{x}{L}$ $\qquad du = \frac{dx}{L}$

$$\int_0^1 \rho(u)\, L\, du = L \int_0^1 \rho(u)\, du$$

$$= 2L$$

$\frac{L>1}{\rho\left(\frac{x}{L}\right)}$



$$\int_0^1 \frac{x\,dx}{x^4+1} \qquad u = x^2$$

$$du = 2x\,dx$$

$$= \int_0^1 \frac{\frac{1}{2}\,du}{u^2+1} = \frac{1}{2}\int_0^1 \frac{du}{u^2+1}$$

$$= \frac{1}{2}\tan^{-1} u \Big|_0^1 = \frac{1}{2}\left(\frac{\pi}{4} - 0\right)$$

$$= \frac{\pi}{8}$$

$$\int \frac{e^x}{\sqrt{1-e^x}}\,dx \qquad u = 1-e^x$$

$$du = -e^x\,dx$$

$$= \int \frac{-du}{\sqrt{u}} = -\int u^{-1/2}\,du$$

$$= -2u^{1/2} + C = -2\sqrt{1-e^x} + C$$

8/29 p 3

$$\int \frac{e^{x/2}\,dx}{\sqrt{1-e^x}}$$

$u = e^{\frac{x}{2}}$

$du = \frac{1}{2} e^{\frac{x}{2}}\,dx$

$$= \int \frac{2\,du}{\sqrt{1-u^2}}$$

$e^x = \left(e^{\frac{x}{2}}\right)^2$
$= u^2$

$$= 2 \sin^{-1}(u) = 2\sin^{-1}\left(e^{\frac{x}{2}}\right) + C$$

7.2 Integration by parts

8/29 p4

$$(uv)' = u'v + uv'$$

$$uv = \int u'v\,dx + \int uv'\,dx$$

$$\int x e^{2x}\,dx$$

$u = x \qquad v' = e^{2x}$

$u' = 1 \qquad v = \frac{1}{2} e^{2x}$

$$= \frac{1}{2} x e^{2x} - \int \frac{1}{2} e^{2x}\,dx$$

$$= \frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x} + C$$

$$\int x^2 e^{2x}\,dx$$

$u = x^2 \qquad v' = e^{2x}$

$u' = 2x \qquad v = \frac{1}{2} e^{2x}$

$$= \frac{1}{2} x^2 e^{2x} - \int 2x \, \frac{1}{2} e^{2x}\,dx$$

$$= \frac{1}{2} x^2 e^{2x} - \left[\frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x} \right] + C$$

$$= \frac{1}{2} x^2 e^{2x} - \frac{1}{2} x e^{2x} + \frac{1}{4} e^{2x} + C$$

Feynmann's trick

Integration by differentiation

$$\int e^{ax}\, dx = \frac{1}{a} e^{ax}$$

$\frac{d}{da}$ this !

$$\int x e^{ax}\, dx = -\frac{1}{a^2} e^{ax} + \frac{x}{a} e^{ax}$$

Set $a = 2$

$$\int x e^{2x}\, dx = -\frac{1}{4} e^{2x} + \frac{x}{2} e^{2x}$$

8/29 p 6

$$\int \ln x \, dx = \int 1 \cdot \ln x \, dx$$

$u = \ln x$, $v' = 1$

$u' = \frac{1}{x}$, $v = x$

$$x \ln x - \int \frac{1}{x} x \, dx$$

$$= x \ln x - x + C$$

$$\int x \ln x \, dx$$

$u = \ln x$ $\quad$ $v' = x$

$u' = \frac{1}{x}$ $\quad$ $v = \frac{1}{2} x^2$

$$\frac{1}{2} x^2 \ln x - \int \frac{1}{x} \frac{1}{2} x^2 \, dx$$

$$= \frac{1}{2} x^2 \ln x - \frac{1}{2} \int x \, dx$$

$$= \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C$$

Circular

$$\int e^{2x} \sin(3x)\,dx$$

$u = \sin(3x) \qquad v' = e^{2x}$

$u' = 3\cos(3x) \qquad v = \frac{1}{2} e^{2x}$

$$= \frac{1}{2}\sin(3x)\, e^{2x} - \frac{3}{2}\int \cos(3x)\, e^{2x}\,dx$$

$u = \cos(3x) \qquad v' = e^{2x}$

$u' = -3\sin(3x) \qquad v = \frac{1}{2} e^{2x}$

$$= \frac{1}{2}\sin(3x)\, e^{2x} - \frac{3}{2}\left[\frac{1}{2}\cos 3x\, e^{2x} - \int (-3)\sin(3x)\,\frac{1}{2} e^{2x}\,dx\right]$$

$$= \frac{1}{2}\sin(3x)\, e^{2x} - \frac{3}{4}\cos(3x)\, e^{2x} - \frac{9}{4}\int \sin(3x)\, e^{2x}\,dx$$

Let $\quad I = \int e^{2x}\sin(3x)\,dx$

$$I = \frac{1}{2}\sin(3x)\, e^{2x} - \frac{3}{4}\cos(3x) e^{2x}$$
$$- \frac{9}{4} I$$

$$\frac{13}{4} I = \dots$$

$$I = \frac{4}{13}\left[\frac{1}{2}\sin(3x)\, e^{2x} - \frac{3}{4}\cos 3x\, e^{2x}\right] + C$$

Use $\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}$

to find $\int_{-\infty}^{\infty} x^2 e^{-x^2}\,dx$

$u = e^{-x^2}$ $\quad v' = x^2$

$u' = -2x\, e^{-x^2}$ $\quad v = \frac{1}{3}x^3$

Bad idea

$u = x \qquad v' = x e^{-x^2}$ | 8/29 p9

$u' = 1 \qquad v = -\frac{1}{2} e^{-x^2}$

$$= -\frac{1}{2} x e^{-x^2} \Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \left(-\frac{1}{2}\right) e^{-x^2} dx$$

$$= 0 - 0 + \frac{1}{2}\sqrt{\pi}$$

$$\int e^{-ax^2} dx = \frac{\sqrt{\pi}}{\sqrt{a}}$$

Use Feymann trick