

§ 1.2 cont

9/10
1

Example

$$\frac{dy}{dx} = \frac{2}{\sqrt{\pi}} e^{-x^2}$$

$$y(0) = 0$$

Solution

$$y(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du$$

$$\frac{dy}{dx} > 0 \quad \text{for all } x$$

Solution $y(x)$ always increases

$$\frac{d^2 y}{dx^2} = \frac{2}{\sqrt{\pi}} e^{-x^2} (-2x)$$

$$< 0 \quad \text{if } x > 0$$

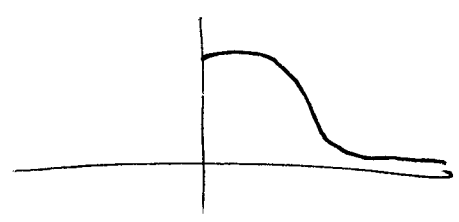
$$> 0 \quad \text{if } x < 0$$

$y(x)$ is concave ~~down~~ if $x > 0$
concave ~~up~~ if $x < 0$

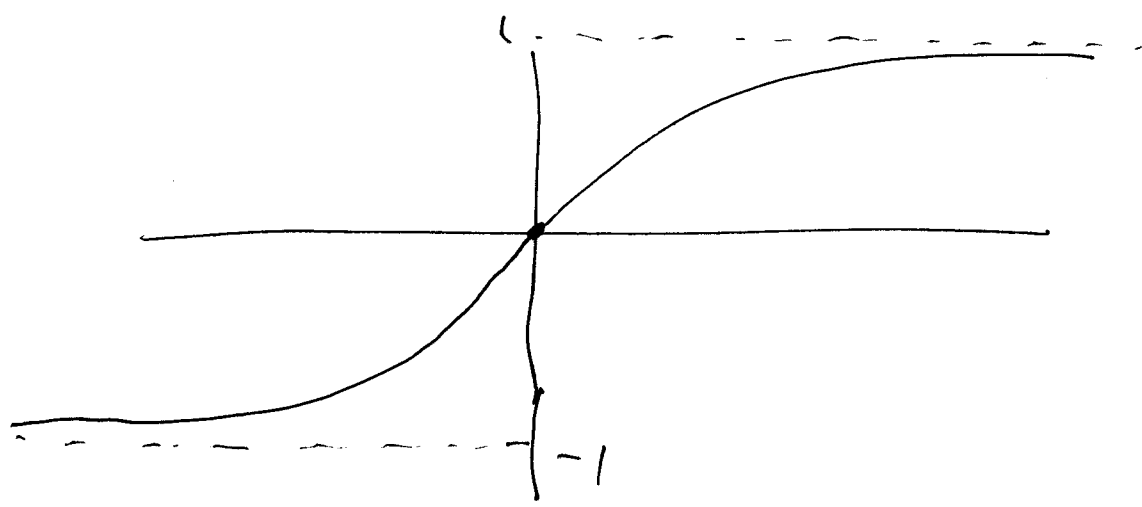
9/10
2

What happens as $a \rightarrow \infty$

$$\frac{2}{\sqrt{\pi}} \int_0^{\infty} e^{-4x^2} dx$$



= 1 (numerical "fact")



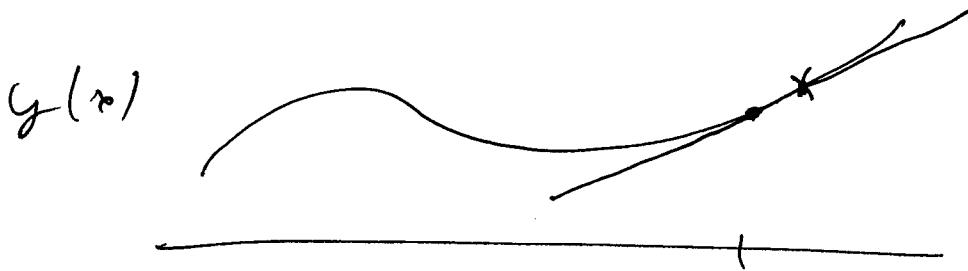
1.3 Slope fields

9/10
3

Good for

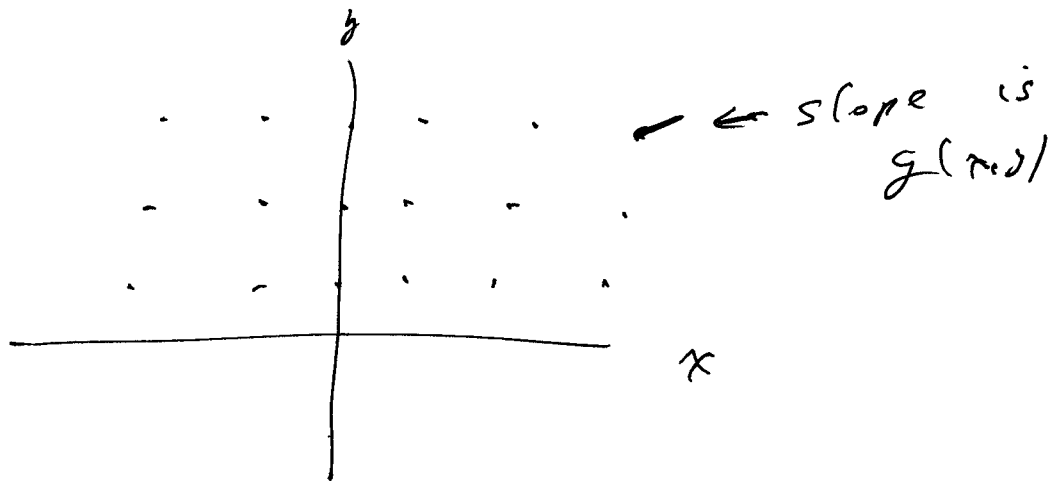
$$\frac{dy}{dx} = g(x, y)$$

Graphical technique



Known $y(x)$

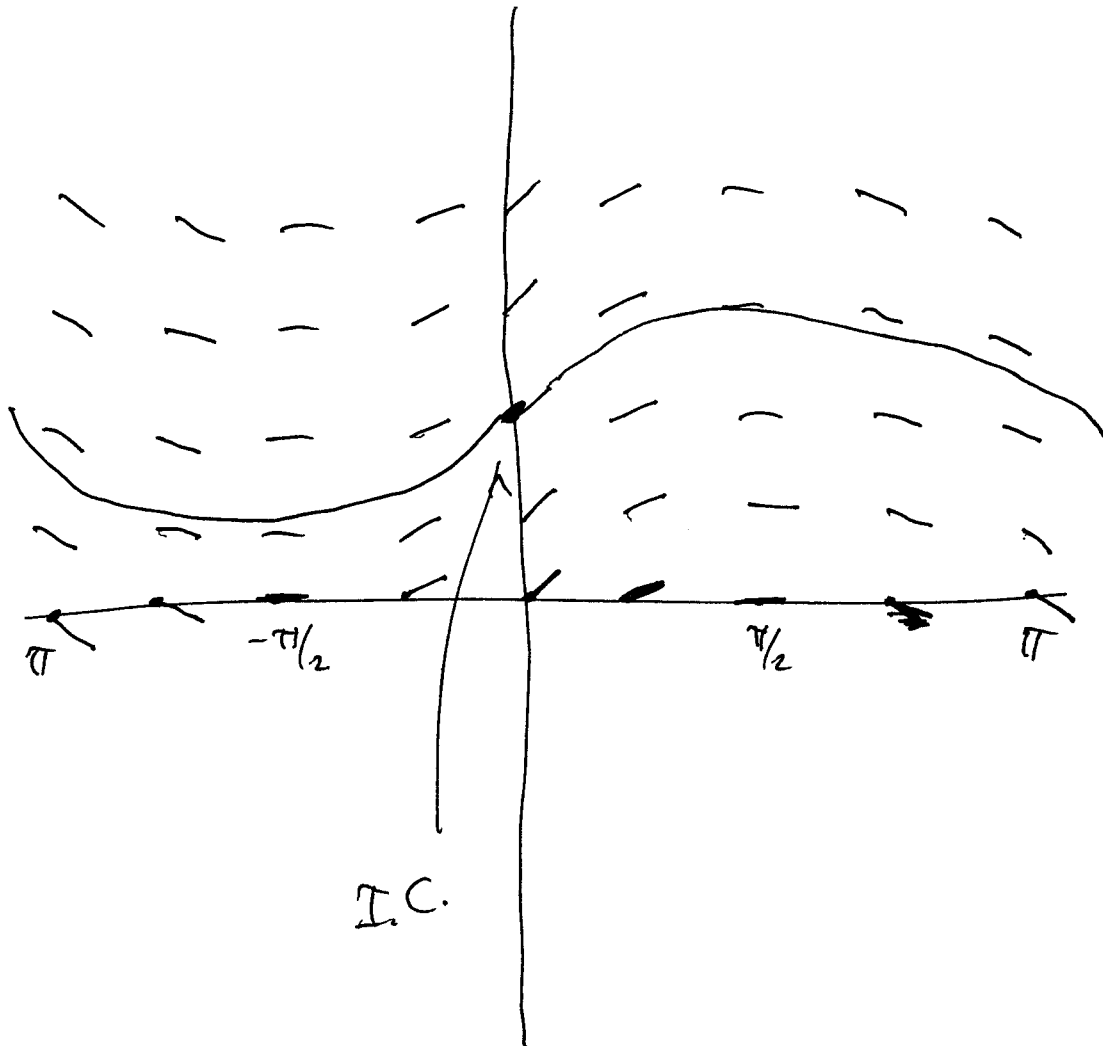
$$y(x + \Delta x) \approx y(x) + \frac{dy}{dx}(x) \Delta x$$



Example

$$g(x,y) = \cos x$$

9/10
4



$$y(x) = \sin x + C$$

ilc class

Go cats 08!

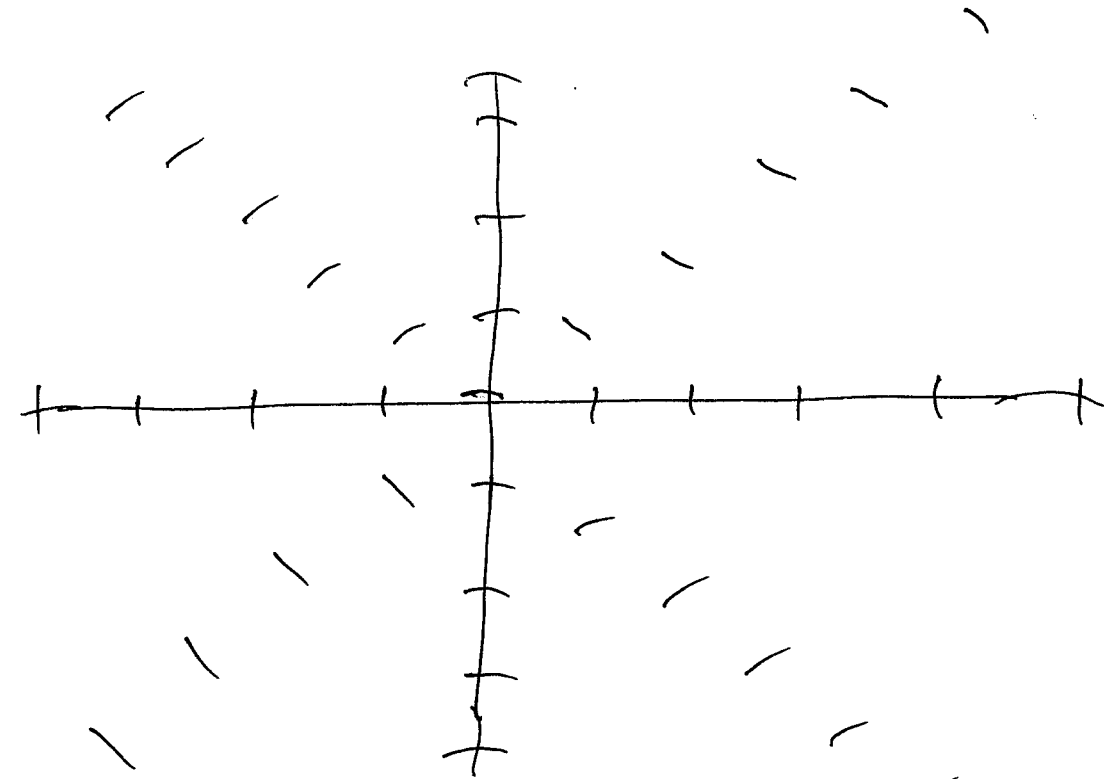
math.arizona.edu/~tgk/250a

$$f(x) = \frac{2}{\sqrt{\pi}} e^{-x^2}$$

$$= 1.12 e^{-x^2}$$

9/10
5

$$\frac{dy}{dx} = -\frac{x}{y}$$



Cook likes

$$y = \sqrt{1-x^2}$$

is a solution

$$x^2 + y^2 = 1$$

9/10
6

Is

$$y = \sqrt{1-x^2}$$

a solution?

$$\frac{dy}{dx} = \frac{1}{2} (1-x^2)^{-1/2} (-2x)$$

$$= \frac{-x}{\sqrt{1-x^2}}$$

equal

$$\frac{-x}{y} = \frac{-x}{\sqrt{1-x^2}}$$

So yes it satisfies dif. eq.