

Chap 1 cont

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①

Uniqueness theorem

$x_0 \in (a, b)$
 x_0 in (a, b)

$$\frac{dy}{dx} = g(x)$$

$g(x)$ is a continuous function on (a, b)

y_1, y_2 are solutions with

~~$y_1(x)$~~

$$y_1(x_0) = y_0$$

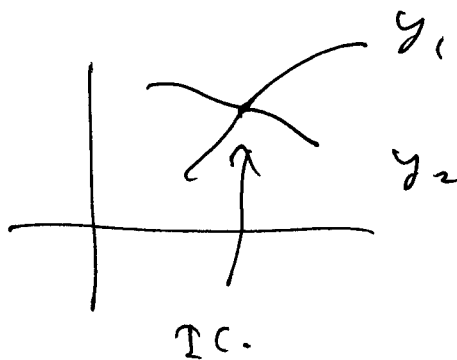
$$y_2(x_0) = y_0$$

, $x_0 \in (a, b)$

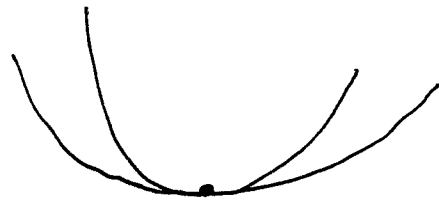
Then $y_1(x) = y_2(x)$ on (a, b)

"Proof"

slope field



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Proof

$$\frac{d}{dx} [y_1(x) - y_2(x)] = f(x) - f(x) = 0$$

$$y_1(x_0) - y_2(x_0) = y_0 - y_0 = 0$$

$$h(x) = y_1(x) - y_2(x)$$

$$\begin{aligned} & h'(x) = 0, \quad h(x_0) = 0 \\ & \hookrightarrow h(x) = \text{constant} \quad \downarrow \\ & \quad \quad \quad \text{constant} = 0 \end{aligned}$$

Chap 2

Autonomous

Dit

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Eq. 3

$$\frac{dy}{dx} = g(y)$$

$$y = y(x)$$

$$\frac{dy}{dx} = \frac{1}{y}, \quad \frac{dy}{dx} = e^y$$

$$y = \int g(y) dx = ?$$

§ 2.1 ~~Def~~ Separation of Variables

Example

$$\frac{dy}{dx} = y^2$$

Calculus: $\frac{dy}{dx} \geq 0 \Rightarrow y$ increasing

$$\begin{aligned} \frac{d^2y}{dx^2} &= 2y \frac{dy}{dx} = 2y y^2 \\ &= 2y^3 \end{aligned}$$

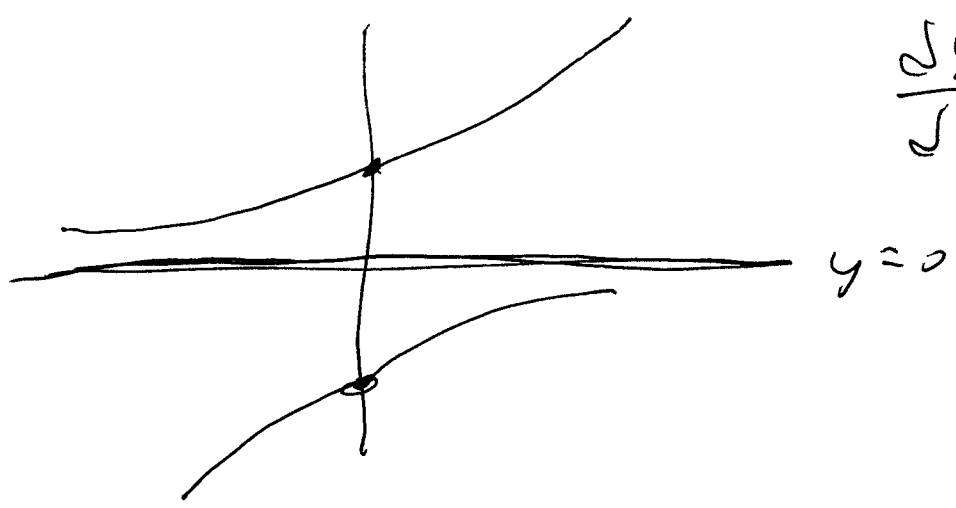
$$y > 0$$

concave up

$$y < 0$$

concave down

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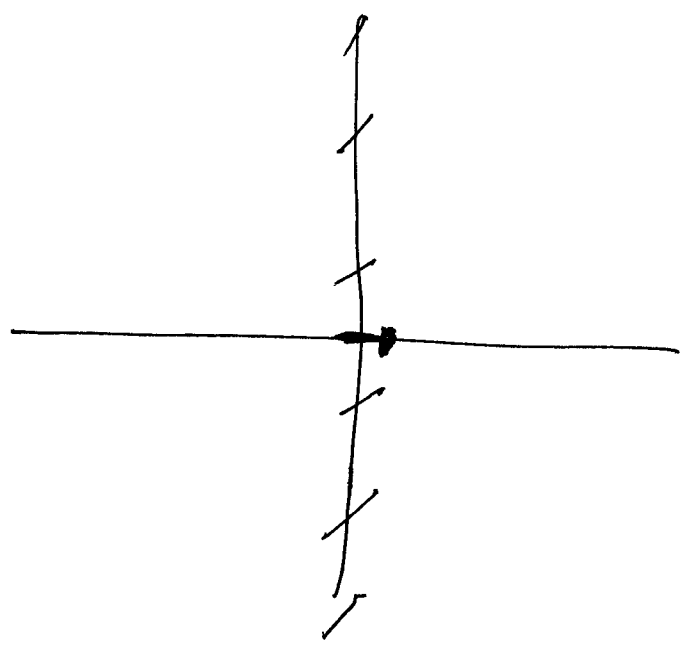


$$\frac{dy}{dx} = y^2$$

Are there constant solutions?

$y = c$ Yes $y = 0$

slope field



Slope is constant in horizontal direction. So $y(x)$ is a solution then $y(x+c)$ is a solution. BUT $y(x) + c$ is usually not.

$$\frac{dy}{dx} = y^2$$

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$$\frac{dy}{y^2} = dx$$

$$\int \frac{dy}{y^2} = \int dx$$

$$C' - y^{-1} = x + C \quad \leftarrow !$$

$$-\frac{1}{y} = x + C$$

$$y = \frac{-1}{x + C}$$

$$\frac{dy}{dx} = \frac{1}{(x+C)^2} = y^2 \quad \text{works!}$$

$$C = 1 \quad y = \frac{-1}{x+1} \quad y(0) = -1$$

$$C = -1 \quad y = \frac{-1}{x-1} \quad y(0) = +1$$

Example

$$\frac{dy}{dx} = ay$$

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a parameter

$$\frac{dy}{y} = a dx$$

$$\int \frac{dy}{y} = \int a dx$$

$$\ln y = ax + C$$

$$y = e^{ax+C}$$

$$y = e^C e^{ax}$$

$$= A e^{ax}$$

$A > 0$

$$\frac{dy}{dx} = A a e^{ax} = ay \quad \text{yeah!}$$

$$\frac{d}{dx} \left(\underbrace{-e^{ax}}_y \right) = -a e^{ax} = ay$$

$$\ln |y| = ax + C$$

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$$|y| = e^{ax+C}$$

$$= A e^{ax}$$

$A > 0$

$$y > 0$$

$$y = A e^{ax}$$

$$y < 0$$

$$-y = A e^{ax}$$

$$y = -A e^{ax}$$

Together:

$$y(x) = A e^{ax}$$

any A

$$|-2| = -(-2) = 2$$