Example $\quad \beta(t)$

$$\frac{d\beta}{dt} = \sin \beta$$

when is $\beta(t) = c$ a solution?

when $\sin c = 0$

$$c = \dots -2\pi, -\pi, 0, \pi, 2\pi \dots$$

A constant solution is called an equilibrium solution.

Such such a solution is stable if for nearby initial conditions the solution ~~B~~ approaches the eq. solution, as $t \to +\infty$.

unstable if nearby solutions move away from it.

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$$\frac{dy}{dx} = g(y)$$

$y = c$ is a eq. solution if and if $g(c) = 0$.

Example

$$\frac{dy}{dt} = c(a-y)y$$

$$\frac{dy}{dt} = -2(3-y)y$$

3 is eq. sol that is unstable

0 is stable

$$\frac{dy}{dt} = 2(3-y)y$$

$$\frac{dy}{dt} = -2(3-y)y$$

$$\frac{dy}{(3-y)y} = -2\,dt$$

$$\int \frac{dy}{(3-y)y} = -2t + C$$

Partial fractions : algebra

$$\frac{1}{(3-y)y} = \frac{A}{3-y} + \frac{B}{y}$$

Find A, B

$$\frac{1}{(3-y)y} = \frac{Ay + B(3-y)}{(3-y)y}$$

$$1 = Ay + B(3-y)$$

$$1 = Ay - By + 3B$$

$\Rightarrow$ $\underset{\text{const}}{1 = 3B}$, $\underset{y\text{-eq.}}{0 = A - B}$

$$1 = 3B, \qquad A - B = 0$$

$$B = 1/3, \qquad A = 1/3$$

$$\frac{1}{(3-y)\,y} = \frac{\frac{1}{3}}{3-y} + \frac{\frac{1}{3}}{y}$$

$$\int \frac{dy}{(3-y)\,y} = \frac{1}{3}\int \frac{dy}{3-y} + \frac{1}{3}\int \frac{dy}{y}$$

$$= -\frac{1}{3}\ln|3-y| + \frac{1}{3}\ln|y|$$

So $-\frac{1}{3}\ln|3-y| + \frac{1}{3}\ln|y| = -2t + C$

$$-\ln|3-y| + \ln|y| = -6t + C'$$

$$e^{-\ln|3-y| + \ln|y|} = e^{C'}e^{-6t}$$

$$\frac{|y|}{|3-y|} = D e^{-6t}, \qquad D > 0$$

Consider cases.