Example

\[
\frac{df}{dt} = \sin f
\]

When is \( f(t) = c \) a solution?

When \( \sin c = 0 \)

\( c = \ldots -2\pi, -\pi, 0, \pi, 2\pi, \ldots \)

A constant solution is called an equilibrium solution.

Such a solution is stable if for nearby initial conditions the solution approaches the eq. solution as \( t \to \infty \).

It is unstable if nearby solution move away from it.
\[
\frac{dy}{dx} = g(y)
\]

\[y = c \quad \text{is a solution if and only if } g(c) = 0.\]

**Example**

\[
\frac{dy}{dt} = c(a - y) y
\]

\[
\frac{dy}{dt} = -2(3-y) y
\]

3 is *c1*, so it is unstable.
0 is stable.

\[
\frac{dy}{dt} = 2(3-y) y
\]
\[ \frac{dy}{dt} = -2 (3-y) y \]
\[ \int \frac{dy}{(3-y) y} = -2 t + C \]

**Partial fractions:** algebra

\[ \frac{1}{(3-y) y} = \frac{A}{3-y} + \frac{B}{y} \]

Find \( A, B \)

\[ \frac{1}{(3-y) y} = \frac{Ay + B (3-y)}{(3-y) y} \]

\[ l = Ay + B (3-y) \]
\[ l = Ay - B y + 3 B \]
\[ \Rightarrow l = 3 B \quad \text{and} \quad 0 = A - B \]
\[ \text{const.} \quad y = e^t. \]
\[ l = 3B, \quad A - B = 0 \]

\[ B = \frac{1}{3}, \quad A = \frac{4}{3} \]

\[ \frac{1}{(3-y)^2} = \frac{1}{3} \frac{1}{3-y} + \frac{1}{y} \]

\[ \int \frac{dy}{(3-y)^2} = \frac{1}{3} \int \frac{dy}{3-y} + \frac{1}{3} \int \frac{dy}{y} \]

\[ = \frac{1}{3} \ln |3-y| + \frac{1}{3} \ln |y| \]

So

\[ -\frac{1}{3} \ln |3-y| + \frac{1}{3} \ln |y| = -2t + C \]

\[ -\ln |3-y| + \ln |y| = -6t + C' \]

\[ e^{-\ln |3-y| + \ln |y|} = e^{C'} e^{-6t} \]

\[ \frac{|y|}{|3-y|} = e^{-6t}, \quad D > 0 \]

Consider cases.