Exam Wed

7.1, 7.2 calc

1.1, 1.2, 1.3 dif eq.

2^{nd} fundamental thm

No "functions with parameters"

No complex numbers

1.3 "due" Wed

2.1 due Fri

Office Hours! Tues 43-5+

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$$\frac{|y|}{|3-y|} = D e^{-6t}, \quad D > 0$$

$|-2| = -(-2)$
$= 2$

Cases $y > 3$

$|y| = y, \quad |3-y| = -(3-y) = y-3$

$$\frac{y}{\cancel{3-y}\; y-3} = D e^{-6t}$$

$$y = D e^{-6t} y - 3 D e^{-6t}$$

$$(1 - D e^{-6t}) y = -3 D e^{-6t}$$

$$y = \frac{-3 D e^{-6t}}{1 - D e^{-6t}}$$

As $t \to \infty$, $e^{-6t} \to 0$

$$y \to \frac{0}{1-0} = 0$$

§ 7.4 Partial fractions

Rational functions

$$\frac{P(x)}{Q(x)} \qquad P, Q \text{ polynomials}$$

If degree of P is $\geq$ degree of Q, do long division.

Example $\dfrac{x^3 + 2x - 3}{x + 2}$

$$
\begin{array}{r|l}
 & \phantom{x^3 + {}} x^2 - 2x + 6 \\
x + 2 & x^3 + 0x^2 + 2x - 3 \\
 & x^3 + 2x^2 \\
\hline
 & -2x^2 + 2x - 3 \\
 & -2x^2 - 4x \\
\hline
 & \phantom{x^3 - 2x^2 + {}} 6x - 3 \\
 & \phantom{x^3 - 2x^2 + {}} 6x + 12 \\
\hline
 & -15
\end{array}
$$

$$\frac{x^3 + 2x - 3}{x + 2} = x^2 - 2x + 6 - \frac{15}{x + 2}$$

Algebra facts

1. $Q(x)$ can be factored into a product of linear terms $(x - c)$ and irreducible quadratic terms.

2. Quadratic polynomial:

one root: $x^2 + bx + c = (x - \alpha)^2$

two roots: $x^2 + bx + c = (x - \alpha)(x - \beta)$

no real roots: (irreducible)

$$x^2 + bx + c = (x - \alpha)^2 + \beta^2$$

(complete the square)

$\dfrac{P(x)}{Q(x)}$ = sum of terms

Examples what to include

(1) $$\frac{8x^2+3x-2}{x^2(x^2+2)(x-5)}$$

$$= \frac{A_1}{x} + \frac{A_2}{x^2} + \frac{Ax+B}{x^2+2} + \frac{C}{x-5}$$

(2) $$\frac{3x^2-2x}{(x-2)(x+2)(x^2+x+10)^2}$$

$$= \frac{A}{x-2} + \frac{B}{x+2} + \frac{Cx+D}{x^2+x+10}$$

$$+ \frac{Ex+F}{(x^2+x+10)^2}$$

$$\cancel{\frac{1}{(x-c)^2} \text{ in } Q(x) \Rightarrow \frac{A_1}{x-c} + \frac{A_2}{(x-c)^2}}$$

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① $\int \frac{2x+1}{(x-1)(x+2)}\,dx$

$$= \int \left(\frac{A}{x-1} + \frac{B}{x+2}\right] dx$$

Algebra

$$\frac{2x+1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$$

$$= \frac{A(x+2) + B(x-1)}{(x-1)(x+2)}$$

So $\quad 2x+1 = A(x+2) + B(x-1)$

$$\underline{2x}+1 = \underline{Ax} + 2A + \underline{Bx} - B$$

$1 = 2A - B \qquad$ (const)

$2 = A + B \qquad$ (x)

add them: $\quad 3 = 3A \qquad A = 1$

$B = 1$

$$\int \frac{2x-1}{(x-1)(x-2)}\,dx = \int \left(\frac{1}{x-1} + \frac{1}{x+2}\right) dx$$

$$= \ln|x-1| + \ln|x+2| + C$$

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$$\int \frac{x^2+5}{x(x^2+1)}\,dx$$

$$\frac{x^2+5}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$$= \frac{A(x^2+1) + (Bx+C)x}{x(x^2+1)}$$

$$x^2+5 = A(x^2+1) + (Bx+C)x$$

$$x^2+5 = Ax^2 + A + Bx^2 + Cx$$

Const: $5 = A$

x : $0 = C$

x^2: $1 = A + B$

$$B = -4$$

$$\int \frac{x^2+5}{x(x^2+1)}\,dx = \int \left(\frac{5}{x} + \frac{-4x}{x^2+1} \right) dx$$

$$= 5\ln|x| - 2\ln(x^2+1) + C$$