

$$\int \frac{1}{x+c} dx = \ln |x+c|$$

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①

$$\int \frac{1}{(x+c)^2} dx = \frac{-1}{x+c}$$

$$\int \frac{Ax+B}{f(x)} dx \quad f(x) \text{ irreducible}$$

Complete the square.

Use sub to turn it into

$$\int \frac{du}{1+u^2} = \tan^{-1} u$$

Example

$$\int \frac{(x-1)}{x^2 + 2x + 5} dx$$

$$= \int \frac{x-1}{(x+1)^2 + 4} dx$$

~~Sub~~ ~~Sub~~ $\int \frac{du}{4u^2 + 4} = \frac{1}{4} \tan^{-1} u$

Sub

$$(x+1)^2 + 4 = 4u^2 + 4$$

$$\left(\begin{array}{l} 4/2 \\ \textcircled{2} \end{array} \right)$$

$$x+1 = 2u$$

$$dx = 2 du$$

$$\cancel{du = \frac{1}{2} dx}$$

$$\int = \int \frac{2u-2}{4u^2+4} 2 du$$

$$= \frac{2}{4} 2 \int \frac{u-1}{u^2+1} du$$

$$= \int \frac{u}{u^2+1} du - \int \frac{du}{u^2+1}$$

$$= \frac{1}{2} \ln(u^2+1) - \tan^{-1} u + C$$

$$= \frac{1}{2} \ln\left(\frac{1}{4}(x+1)^2 + 1\right) - \tan^{-1}\left(\frac{x+1}{2}\right) + C$$

$$\int \frac{dx}{\sqrt{-x^2 + 6x - 5}}$$

$$= \int \frac{dx}{\sqrt{-(x-3)^2 + 4}}$$

$$\int \frac{du}{\sqrt{1-u^2}}$$

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③

$$\int \frac{du}{\sqrt{4-4u^2}}$$

$$(x-3)^2 = 4u^2$$

$$x-3 = 2u$$

$$dx = 2du$$

$$= \int \frac{2du}{\sqrt{4-4u^2}} = \int \frac{du}{\sqrt{1-u^2}}$$

$$= \sin^{-1}\left(\frac{u}{1}\right) + C = \sin^{-1}\left(\frac{x-3}{2}\right) + C$$

$$-x^2 + 6x - 5 = -(x^2 - 6x + 5)$$

$$= -((x-3)^2 - 4)$$

$$= -(x-3)^2 + 4$$

2.2 cont

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④

Pop of ~~Tucson~~ Tucson $P(t)$
satisfies

$$P'(t) = k P(t)$$

General sol $P(t) = C e^{kt}$

2000 720,000

2005 780,000

$t=0$ is year 2000

$$P(0) = 720,000$$

$$P(0) = C e^0 = C$$

$$C = 720,000$$

$$P(5) = 780,000$$

$$C e^{5k} = 780,000$$

$$e^{5k} = \frac{780,000}{720,000} = \frac{78}{72}$$

$$5k = \ln \frac{78}{72}$$

$$k = \frac{\ln \frac{78}{72}}{5}$$

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⑤

Doubling

$$P(t_D) = 2P(0)$$

$$C e^{kt_D} = 2C e^0$$

$$e^{kt_D} = 2$$

$$kt_D = \ln 2$$

$$t_D = \frac{\ln 2}{k}$$