Example that wouldn't see

\[ \frac{dy}{dt} = -2y (3-y) \]

\[ \frac{141}{3-y} = b \ e^{-6t} \]

\( y > 3 \)

\[ \frac{y}{y-3} = D \ e^{-6t} \]

T.C. \( y(0) = 4 \)

\[ \frac{4}{4-3} = 0 \ e^{-0} \]

\( D = 4 \)

\[ \frac{y}{y-3} = 4 \ e^{-6t} \]

\[ y = 4 e^{-6t} \ y - 3 \ 2 e^{-6t} \]
\[ y = -4e^{-6t}, \quad y = -3e^{-6t} \]

\[ y = \frac{240e^{-6t}}{1 - 4e^{-6t}} \]

\[ t = 0 \quad \Rightarrow \quad \frac{-12}{-4} = \frac{-12}{-3} = 4 \]

\[ t \geq 1 \]

At some \( t \), \[ 1 - 4e^{-6t} = 0 \]

\[ e^{-6t} = \frac{1}{4} \]

\[ e^{6t} = 4 \]

\[ 6t = \ln 4 \]

\[ t = \frac{\ln 4}{6} \quad \text{blow up!} \]

\[ \text{valid only for} \quad t < \frac{\ln 4}{6} \]
Population growth

\[ y(t) = \text{population} \]

Simplest model (2.2)

\[ \frac{dy}{dt} = ky, \quad k > 0 \]

\[ y(t) = y(0) e^{kt} \]

\[ g(y) \]

\[ ky \]

\[ \frac{dy}{dt} = g(y) \]

Suppose the environment can only sustain a population of \[ y = b \]

\[ g(y) \]

\[ a y (b - y) \]
\[
\frac{dy}{dt} = ay(b-y)
\]
\[b > 0 \quad a > 0\]

\[y \geq 0\]

Solve it

\[
\frac{dy}{y(b-y)} = adt
\]

\[
\frac{1}{y(b-y)} = \frac{A}{y} + \frac{B}{b-y}
\]

\[= \frac{A(b-y) + By}{y(b-y)}\]

\[A \mid_{b-y} + By = 1\]

\[-Ay + By + Ab = 1\]

\[-A + B = 0\]

\[A \neq 0, \quad Ab = 1\]

\[B = \frac{1}{b}\]

\[A = \frac{1}{b}\]
\[
\frac{1}{b} \ln \left| \frac{y}{b-y} \right| = \frac{1}{b} \ln \left| b-y \right| = at + c
\]

\[
\ln \left| \frac{y}{b-y} \right| = abt + c'
\]

\[
\frac{y}{b-y} = D e^{abt}
\]

\[
y = b D e^{abt} - y D e^{abt}
\]

\[
y + y D e^{abt} = b D e^{abt}
\]

\[
y(t) = \frac{b D e^{abt}}{1 + D e^{abt}} = \frac{b D}{e^{-abt} + D}
\]

\[
y(0) = \frac{60}{1 + D}
\]
\$ 2.4 \textbf{Existence / Uniqueness} \$

\underline{Local existence}

\[
\frac{dy}{dt} = g(x, y), \quad y(t_0) = y_0
\]

Is there a solution which exists for \( t \) near \( t_0 \)?

\underline{Global existence}

Is there a solution which exists for all \( t \), \( -\infty < t < \infty \)?

Prove global existence for the Navier-Stokes equations

\( \Rightarrow \) \( \$(1,000,000) \)
Uniqueness

\[
\frac{dy}{dt} = g(x, y), \quad y(t_0) = y_0
\]

Is there more than one solution \((t_0, y_0)\)?