Example

\[
\frac{dy}{dx} = y^2 = g(x, y)
\]

\[
\frac{dg}{dy} = 2y
\]

This applies \Rightarrow local existence uniqueness

Equilibrium solution:

\[g(x) = 0\]

No other solution can intersect the graph of \(0\).

\[\text{Not allowed!}\]
If solution is positive at some $x$, it is always positive.
Example

\[ \frac{dy}{dx} = y^{2/3} = f(x,y) \]

\[ \frac{2y}{2} = \frac{2}{3} y^{-1/3} \not= \text{ not defined at 0} \]

Theorem does not apply for initial condition with \( y = 0 \)

\[ \frac{dy}{y^{2/3}} = dx \Rightarrow y^{-2/3} dy = dx \]

\[ 3 y^{1/3} = x + C \]

\[ y^{1/3} = \frac{x + C}{3} \]

Equilibrium: \( y = 0 \)

But also solution:

\[ y = \left( \frac{x}{3} \right)^3 \]
Two solutions through \((0, 0)\):
\[ y(x) = 0 \]
\[ y(x) = \left(\frac{x}{3}\right)^3 \]
No uniqueness.