

Logic

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①

If A then B

Suppose A is false.

Then says nothing

~~dy~~
~~dx~~

$$\frac{dy}{dx} = g(x) \leftarrow \text{continuous}$$

as in chap. 1

Then applies, so local existence
& uniqueness

Example

$$\frac{dy}{dx} = \sqrt{1-y^2}$$

Read book

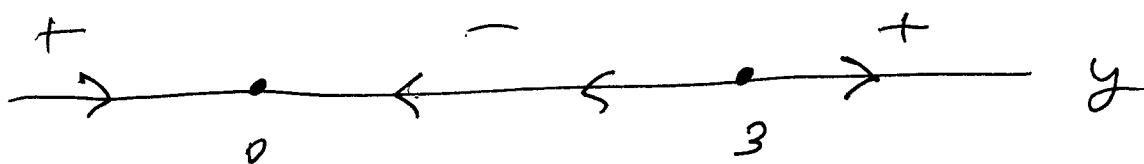
§ 2.5 phase lines 9/28
①

Warm up for "phase plane analysis"

Example

$$y' = y^2 - 3y = y(y-3)$$

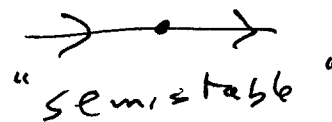
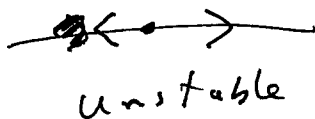
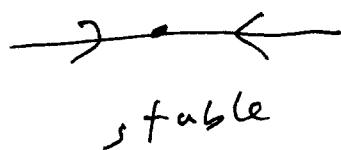
Equilibria are $y = 0$
 $y = 3$



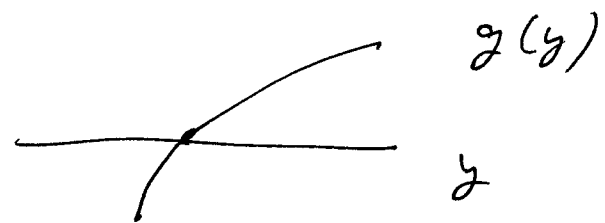
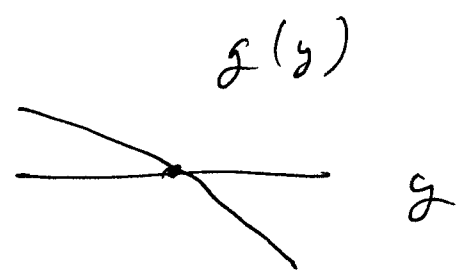
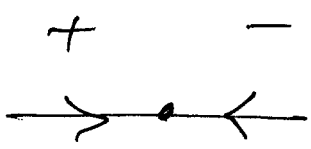
Determine signs of $y(y-3)$

0 is stable

3 is unstable

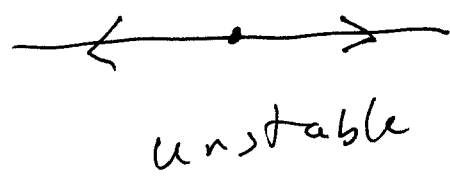
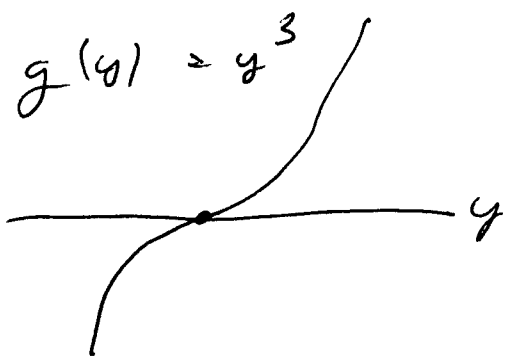


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 (2)



$g'(c) = 0$

\Rightarrow nothing



$\frac{dy}{dt} = g(y)$

Example

$$= 8y - 2y^2$$

$$\frac{9}{28} \textcircled{3}$$

$$\frac{dy}{dx} = 2y(4-y) \leftarrow g(y)$$

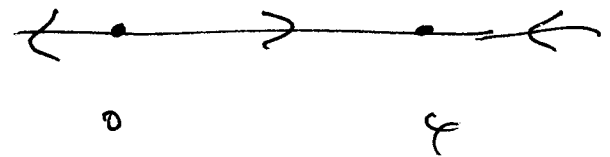
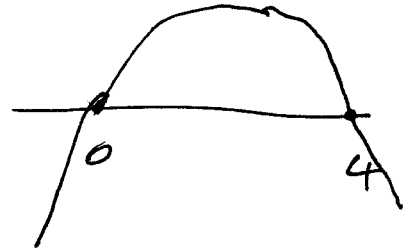
$$g' = 8 - 4y$$

$$g'(0) = 8$$

0 is unstable

$$g'(4) = -8$$

4 is stable



Example

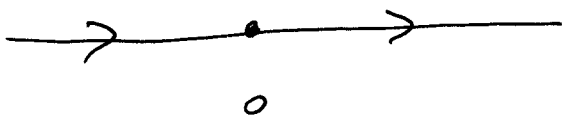
$$\frac{dy}{dx}$$

$$= y^2 \leftarrow g$$

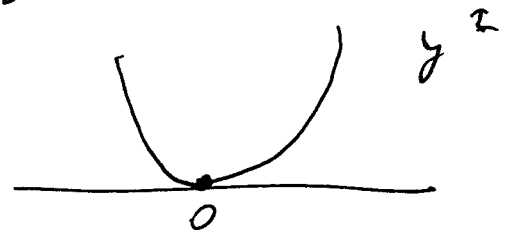
0 is only equilibrium

$$g'(y) = 2y$$

$$g'(0) = 0$$



semistable



Example

$$\frac{dR}{dt} = \sin(\pi R) \leftarrow g(R)$$

$g(R)$
④

Equilibria: $R = \dots, -2, -1, 0, 1, 2, \dots$

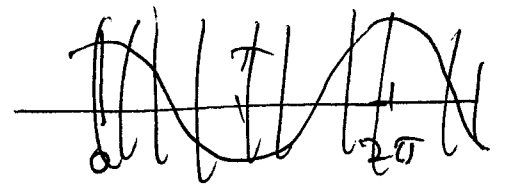
$$g'(R) = \pi \cos(\pi R)$$

$g'(0) > 0$ unstable

$g'(1) < 0$ stable

$g'(2) > 0$ unstable

$g'(3) < 0$



$2n$ is unstable

$2n+1$ is stable

