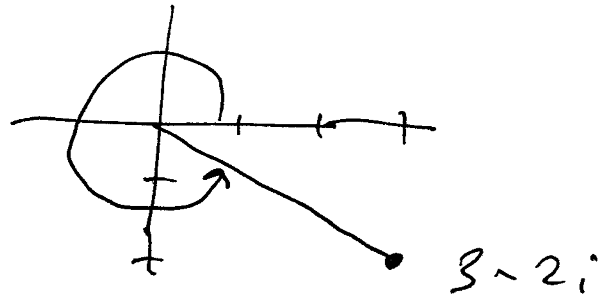
Polar coordinates

$x + iy$

Plot it as (x, y)

$3 - 2i$



Find polar coordinates of (x, y)

$x = r\cos\theta$

$y = r\sin\theta$

Then $x + iy = re^{i\theta}$

$\tan\theta = \frac{y}{x}$, $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

9/5
2

$$e^{i\theta} = \cos\theta + i\sin\theta \quad \leftarrow$$

$$e^{-i\theta} = \cos(-\theta) + i\sin(-\theta)$$

$$e^{-i\theta} = \cos(\theta) - i\sin(\theta) \quad \leftarrow$$

Add eqs:

$$e^{i\theta} + e^{-i\theta} = 2\cos\theta$$

Subtract eqs:

$$e^{i\theta} - e^{-i\theta} = 2i\sin\theta$$

Integration

$$\int \sin\theta \, d\theta = \int \frac{e^{i\theta} - e^{-i\theta}}{2i} \, d\theta$$

$$= \frac{1}{2i}\left[\frac{1}{i}e^{i\theta} - \frac{1}{-i}e^{-i\theta}\right]$$

$$= \frac{1}{2}\left[-e^{i\theta} - e^{-i\theta}\right] = -\cos\theta$$

9/5
3

$\boxed{\frac{1}{i} = -i}$

$$\int \sin^2\theta \, d\theta$$

$$= \int \left(\frac{e^{i\theta} - e^{-i\theta}}{2i}\right)^2 d\theta$$

$$= -\frac{1}{4} \int \frac{e^{2i\theta} - 2 + e^{-2i\theta}}{} \, d\theta$$

$$= -\frac{1}{4}\left[\frac{1}{2i} e^{2i\theta} - 2\theta + \frac{1}{-2i} e^{-2i\theta}\right]$$

$$= \frac{1}{2}\cancel{\theta} + \frac{1}{8}\left(i \cancel{e^{2i\theta}} + \cancel{i} \cancel{e^{-2i\theta}}\right)$$

$$= \frac{-1}{8i} e^{2i\theta} + \frac{1}{2}\theta + \frac{1}{8i} e^{-2i\theta}$$

$$= \frac{1}{2}\theta + \frac{-e^{2i\theta} + e^{-2i\theta}}{8i}$$

$$= \frac{1}{2}\theta + \frac{-\sin(2\theta)}{4} + C$$

$$= \frac{1}{2}\theta - \frac{1}{4}\sin(2\theta) + C$$

9/5
4

Find

$$\int e^{2x} \sin(3x)\, dx$$

$$= \int e^{2x} \frac{e^{3ix} - e^{-3ix}}{2i}\, dx$$

$$= \frac{1}{2i} \int \left(e^{2x+3ix} - e^{2x-3ix} \right) dx$$

$$= \frac{1}{2i} \int \left[e^{(2+3i)x} - e^{(2-3i)x} \right] dx$$

$$= \frac{1}{2i} \left[\frac{1}{2+3i} e^{(2+3i)x} - \frac{1}{2-3i} e^{(2-3i)x} \right]$$

more algebra to do

$$= \frac{1}{2i} \left[\frac{1}{2+3i} e^{2x} (\cos 3x + i \sin 3x) - \frac{1}{2-3i} e^{2x} (\cos 3x - i \sin 3x) \right]$$

Side

$$e^{(2+3i)x} = e^{2x} e^{3ix}$$

$$= e^{2x} (\cos 3x + i \sin 3x)$$

Dif Eq.

9/15
5

$$\frac{dy}{dx} = x^2$$

$$\frac{dy}{dx} = x^3 + \sin y$$

$$\frac{dy}{dx} = \sin(xy)$$

$$\frac{dP}{dt} = e^{tP}$$

First order

second order

$$\frac{d^2y}{dx^2} + \sin x \frac{dy}{dx} + 6y = 0$$

$$F = ma \qquad F = m\frac{d^2x}{dt^2}$$

9/5
6

First order dif. eq.

is of the form

$$\frac{dy}{dx} = g(x, y)$$

$$y(x_0) = y_0$$

e.g. $y(0) = 7$

Chap 1

$$\frac{dy}{dx} = g(x)$$

$\Leftrightarrow$ integrating $g(x)$

Chap 2

$$\frac{dy}{dx} = g(y)$$

Chap 3

$$\frac{dy}{dx} = g(x, y)$$

1) analytic 2) graphical 3) numerical

1.1 Simple DE

9/5

7

Examples Find $y(x)$ such that

$$\frac{dy}{dx} = \sin x \quad , \quad y(0) = 2$$

$y(x) = -\cos x + C$

$y(0) = -\cos(0) + C = -1 + C$

So need $-1 + C = 2$

$C = 3$

Solution is $y(x) = -\cos x + 3$

9/5

8

2. Find $y(t)$ such that

$$\frac{dy}{dt} = e^{5t}$$

$$y(1) = 10$$

$$y(t) = \frac{1}{5} e^{5t} + C$$

$$y(1) = \frac{1}{5} e^{5} + C$$

so $\frac{1}{5} e^{5} + C = 10$

$$C = 10 - \frac{1}{5} e^{5}$$

$$y(t) = \frac{1}{5} e^{5t} + 10 - \frac{1}{5} e^{5}$$