1. (a) Solve the following linear differential equation and initial condition:

\[ x y' + y = x \sin(x), \quad y(\pi) = 1 \]

\[ y' + \frac{1}{x} y = \sin(x) \quad p = \frac{1}{x} \]

\[ x y' + y = x \sin(x) \quad e^{\int p \, dx} = e^{\int \frac{1}{x} \, dx} = x \]

\[ (xy)' = x \sin(x) \]

\[ x y = \int x \sin(x) \, dx \]

\[ u = x, \quad dv = \sin(x) \, dx \]

\[ du = dx, \quad v = -\cos(x) \]

\[ x y = -x \cos(x) + \int \cos(x) \, dx \]

\[ x y = -x \cos(x) + \sin(x) + C \]

\[ \pi = -\pi \cos(\pi) + \sin(\pi) + C \]

\[ \cos(\pi) = -1 \quad \text{so} \quad C = 0 \]

\[ y = -\cos(x) + \frac{\sin(x)}{\pi} \]

(b) Suppose that \( x \) is time. Identify the transient and steady state parts of your solution.

\[ \text{As } x \to \infty, \quad \frac{\sin(x)}{x} \to 0 \]

So \( \frac{\sin(x)}{x} \) is transient part

\[-\cos(x) \] is steady state part