

Math 250a (Kennedy) - Quiz 10 - Fall '07

(16 points)

1. (a) Solve the following linear differential equation and initial condition:

$$xy' + y = x \sin(x), \quad y(\pi) = 1$$

$$y' + \frac{1}{x} y = \sin x$$

$$p = \frac{1}{x}$$

$$xy' + y = x \sin x$$

$$e^{\int p} = e^{\int \frac{1}{x}} = x$$

$$(xy)' = x \sin x$$

$$xy = \int x \sin x \, dx$$

$$u = x \quad dv = \sin x \, dx$$

$$du = dx \quad v = -\cos x$$

$$xy = -x \cos x + \int \cos x \, dx$$

$$xy = -x \cos x + \sin x + C$$

$$\pi = -\pi \cos \pi + \sin \pi + C$$

$$\cos \pi = -1 \quad \text{so} \quad C = 0$$

$$y = -\cos x + \frac{\sin x}{x}$$

(4 points)

(b) Suppose that x is time. Identify the transient and steady state parts of your solution.

As $x \rightarrow \infty$, $\frac{\sin x}{x} \rightarrow 0$

So $\frac{\sin x}{x}$ is transient part

$-\cos x$ is steady state part