1. Find the solution of each of these diff. eqs. with initial conditions.
   
   (a) \[ \frac{df}{dt} = e^{-2t}, \quad f(0) = 0 \]

   \[ f(t) = -\frac{1}{2} e^{-2t} + C \]

   \[ f(0) = -\frac{1}{2} + C \]

   So \[ C = \frac{1}{2} \]

   \[ f(t) = -\frac{1}{2} e^{-2t} + \frac{1}{2} \]

   
   (b) \[ \frac{dy}{dx} = \frac{x}{1 + x^2}, \quad y(0) = 2 \]

   Use \textit{sub} \[ u = 1 + x^2 \] or guess

   \[ \int \frac{x}{1 + x^2} \, dx = \frac{1}{2} \ln(1 + x^2) + C \]

   So \[ y(x) = \frac{1}{2} \ln(1 + x^2) + C \]

   \[ y(0) = \frac{1}{2} \ln(1) + C = 2 \]

   So \[ C = 2 \]

   \[ y(x) = \frac{1}{2} \ln(1 + x^2) + 2 \]
2. Consider the diff. eq. and initial condition \( \frac{dy}{dx} = x^2 e^{-2x^2}, \quad y(0) = 1 \)

(a) Determine where the solution is increasing, decreasing, concave up and concave down.

\[
\chi^2 e^{-2 \chi^2} \geq 0 \quad \text{for all } \chi.
\]

So \( y(\chi) \) is always increasing.

\[
\frac{d^2 y}{dx^2} = 2 \chi e^{-2 \chi^2} - 4 \chi^2 e^{-2 \chi^2} = 2 \chi (1 - 2 \chi^2) e^{-2 \chi^2}
\]

Look at sign of \( \chi (1 - 2 \chi^2) \)

\[
\begin{array}{c|c|c|c|c}
+ & - & 0 & + & - \\
\hline
\text{Concave} & \text{up} & \text{down} & \text{up} & \text{down}
\end{array}
\]

(b) Write down an integral that represents the solution.

\[
y(\chi) = \int_0^\chi u^2 e^{-2u^2} du + 1
\]

(c) Given that \( \int_0^\infty x^2 e^{-2x^2} dx = \frac{\sqrt{\pi}}{16} \), find the horizontal asymptote of the solution as \( x \to \infty \).

\[
y(\chi) \to \int_0^\infty u^2 e^{-2u^2} du + 1
\]

\[
= 1 + \frac{\sqrt{\pi}}{16}
\]