

Math 250b (Spring '08) - Homework 7

1. Consider the first order system of differential equations.

$$\frac{dx}{dt} = -4y, \quad \frac{dy}{dt} = 4x$$

- (a) Use the computer to graph the phase portrait and give qualitative descriptions of the solutions.
(b) The solutions are of the form

$$x(t) = C \cos(\omega t + \phi), \quad y(t) = C \sin(\omega t + \phi)$$

Find conditions on C, ω, ϕ which make this a solution.

2. Consider the first order system of differential equations.

$$\frac{dx}{dt} = -4y(x^2 + y^2), \quad \frac{dy}{dt} = 4x(x^2 + y^2)$$

- (a) Use the computer to graph the phase portrait and give qualitative description of the solutions. In particular, how are they different from those in the previous problem?
(b) The solutions are of the form

$$x(t) = C \cos(\omega t + \phi), \quad y(t) = C \sin(\omega t + \phi)$$

Find conditions on C, ω, ϕ which make this a solution.

3. This is a continuation of problem 1 from section 6.3. For the second order dif. eq. in each of the five parts that were assigned, find the corresponding first order system. Use the computer to plot the phase portrait of the first order system and explain how it agrees with the analytic solution you found before.