Math 250b (Kennedy) - Quiz 3 - Spring '08

1. For each of the following series, determine whether it converges or diverges. You should explain your reasoning; in particular cite the test that you use.

$$\sum_{n=1}^{\infty} \frac{n^2}{3^n}$$

Use ratio test:

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)^2 3^n}{3^{n+1} n^2} = \frac{(n+1)^2}{3n^2} \to \frac{1}{3}$$

Since this is less than 1, the ratio test says the series converges.

$$\sum_{n=3}^{\infty} \frac{1}{n^2 \ln(n)}$$

Use comparison test.

$$\frac{1}{n^2 \ln(n)} \leq \frac{1}{n^2}$$

We know that $\sum_n 1/n^2$ converges (integral test), so the given series converges by the comparison test.

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{n(n+1)}$$

This is an alternating series. Since $\frac{1}{n(n+1)}$ decreases and converges to 0 , the alternating series test says the series conveges.

Another approach is to show the series converges absolutely by the comparion test and so the series converges.

$$\sum_{n=1}^{\infty} \left[1 + \frac{1}{2^n}\right]$$

Since $1 + \frac{1}{2^n}$ conveges to 1, by the "stupid test" the series does not converge.