

Math 250b (Kennedy) - Quiz 4 Solutions - Spring '08

1.

$$\sum_{n=1}^{\infty} \frac{x^{2n}}{n!}$$

Ratio test:

$$\frac{x^{2(n+1)}}{(n+1)!} \times \frac{n!}{x^{2n}} = \frac{x^2}{n+1}$$

This converges to 0 as $n \rightarrow \infty$ for all x , so the series converges for all x .

$$\sum_{n=1}^{\infty} \frac{(x+2)^n n}{3^n}$$

Ratio test:

$$\frac{|x+2|^{n+1} (n+1)}{3^{n+1}} \times \frac{3^n}{|x+2|^n n} = \frac{|x+2|}{3} \frac{n+1}{n}$$

This converges to $|x+2|/3$. So the series converges if $|x+2|/3 < 1$, i.e., $|x+2| < 3$. This is the interval $(-5, 1)$.

2. Find the third order Taylor polynomial for $f(x) = \ln(x)$ about $a = 2$.

$$\begin{aligned} f(x) &= \ln(x), & f(2) &= \ln(2) \\ f'(x) &= \frac{1}{x}, & f'(2) &= \frac{1}{2} \\ f''(x) &= \frac{-1}{x^2}, & f''(2) &= \frac{-1}{4} \\ f^{(3)}(x) &= \frac{2}{x^3}, & f^{(3)}(2) &= \frac{1}{4} \end{aligned}$$

So third order Taylor polynomial is

$$\begin{aligned} &f(2) + f'(2)(x-2) + \frac{f''(2)}{2!}(x-2)^2 + \frac{f^{(3)}(2)}{3!}(x-2)^3 \\ &= \ln(2) + \frac{1}{2}(x-2) - \frac{1}{8}(x-2)^2 + \frac{1}{24}(x-2)^3 \end{aligned}$$