Math 250b (Kennedy) - Quiz 5 Solutions - Spring '08

1. Find the solution of each of the following second order differential equations with initial conditions.
(a)

$$x'' - 3x' + 2x = 0, \quad x(0) = 1, \quad x'(0) = 0$$

Characteristic equation is $r^2 - 3r + 2 = 0$. This has roots $r = 1, 2$. So the general solution is

$$x(t) = c_1 e^{2t} + c_2 e^t$$

$$x'(t) = 2c_1 e^{2t} + c_2 e^t$$

So the initial conditions implies

$$x(0) = 1 \quad \Rightarrow \quad c_1 + c_2 = 1$$

$$x'(0) = 1 \quad \Rightarrow \quad 2c_1 + c_2 = 0$$

Solving these two equations we find $c_1 = -1$, $c_2 = 2$. So

$$x(t) = 2e^t - e^{2t}$$

(b)

$$x'' - 4x' + 5x = 0, \quad x(0) = 0, \quad x'(0) = 1$$

Characteristic equation is $r^2 - 4r + 5 = 0$. This has roots $r = 2 \pm i$. So the general solution is

$$x(t) = c_1 e^{2t} \cos(t) + c_2 e^{2t} \sin(t)$$

So $x(0) = c_1$. The initial condition $x(0) = 0$ implies $c_1 = 0$. So

$$x(t) = c_2 e^{2t} \sin(t)$$

and so

$$x'(t) = 2c_2 e^{2t} \sin(t) + c_2 e^{2t} \cos(t)$$

Thus $x'(0) = c_2$. The initial condition $x'(0) = 1$ then implies $c_2 = 1$. So

$$x(t) = e^{2t} \sin(t)$$