Math 250b (Kennedy) - Quiz 6 - Spring ’08

1. Consider the first order system

\[
\begin{align*}
x' &= y \\
y' &= 3x^2 - 3
\end{align*}
\]

(a) Find a second order differential equation for \(x\) that corresponds to this system.

**Solution:** \(x'' = y' = 3x^2 - 3\) So second order equation for \(x\) is \(x'' = 3x^2 - 3\) or \(x'' - 3x^2 + 3 = 0\).

(b) Find the equilibrium solutions of the first order system.

**Solution:** Equilibrium means \(x' = 0\) and \(y' = 0\). So \(y = 0\) and \(3x^2 = 3\). These equations have two solutions: \((1, 0)\) and \((-1, 0)\).

(c) Use the phase plane of the system to give yes or no answers to the following (no explanation required)

- Are there any periodic solutions? **YES**
- Are there any unbounded solutions? **YES**
- Are there any bounded solutions which are not periodic? **NO**

(d) Find an equation for the trajectories in the \(x\)-\(y\) plane.

**Solution:**

\[
\frac{dy}{dx} = \frac{y'}{x'} = \frac{3x^2 - 3}{y}
\]

Solve by separation of variables:

\[
y \, dy = (3x^2 - 3) \, dx
\]

\[
\frac{1}{2}y^2 = x^3 - 3x + C
\]

You can leave the equation like this or solve for \(y\):

\[
y = \pm \sqrt{2x^3 - 6x + 2C}
\]

Different choices for \(C\) give different trajectories.