

Math 250b (Kennedy) - Quiz 6 - Spring '08

1. Consider the first order system

$$\begin{aligned}x' &= y \\y' &= 3x^2 - 3\end{aligned}$$

(a) Find a second order differential equation for x that corresponds to this system.

Solution: $x'' = y' = 3x^2 - 3$ So second order equation for x is $x'' = 3x^2 - 3$ or $x'' - 3x^2 + 3 = 0$.

(b) Find the equilibrium solutions of the first order system.

Solution: Equilibrium means $x' = 0$ and $y' = 0$. So $y = 0$ and $3x^2 = 3$. These equations have two solutions: $(1, 0)$ and $(-1, 0)$.

(c) Use the phase plane of the system to give yes or no answers to the following (no explanation required)

Are there any periodic solutions? **YES**

Are there any unbounded solutions? **YES**

Are there any bounded solutions which are not periodic? **NO**

(d) Find an equation for the trajectories in the x - y plane.

Solution:

$$\frac{dy}{dx} = \frac{y'}{x'} = \frac{3x^2 - 3}{y}$$

Solve by separation of variables:

$$y \, dy = (3x^2 - 3) \, dx$$

$$\frac{1}{2}y^2 = x^3 - 3x + C$$

You can leave the equation like this or solve for y :

$$y = \pm\sqrt{2x^3 - 6x + 2C}$$

Different choices for C give different trajectories.