

Math 466 - Homework 1

1. Consider the experiment of throwing a fair die n times. Let X_1, X_2, \dots, X_n be the results. (So X_i can be 1, 2, 3, 4, 5 or 6.) Let

$$Y = \sum_{i=1}^n X_i \tag{1}$$

Find the following:

- (a) the mean and variance of X_i
- (b) the mean and variance of Y
- (c) the mean and variance of Y/n .

2. Let Z be a standard normal random variable (standard means it has mean zero and variance one.) Let $X = Z^2$. Find the probability density function of X . Hint: the standard trick is to first compute the cumulative distribution function $F(t) = P(X \leq t)$. Then remember that the probability density function is obtained from $f(x) = F'(x)$.

3. Let X_1 and X_2 be independent random variables, each of which has a Poisson distribution with $E[X_i] = \lambda_i$.

- (a) Use the moment generating function to compute $E[X_1^3]$ and $E[X_1^2 X_2^2]$.
- (b) Use the moment generating function to show that $X_1 + X_2$ has a Poisson distribution.

4. (a) For a standard normal random variable the moment generating function is $\phi(t) = \exp(t^2/2)$. Derive this formula.

(b) Let X_1, X_2 be independent normal random variables with means μ_1, μ_2 and standard deviations σ_1, σ_2 . Use moment generating functions to show that $X_1 + X_2$ is a normal random variable and find its mean and standard deviation.

5. I flip a fair coin 1000 times. X is the number of heads I get.

- (a) Find the mean and variance of X .
- (b) Use the central limit theorem to find (approximately) the probabilities that $X \geq 510$, $X \geq 550$, $X \geq 600$.