1. Consider the experiment of throwing a fair die \( n \) times. Let \( X_1, X_2, \ldots, X_n \) be the results. (So \( X_i \) can be 1, 2, 3, 4, 5 or 6.) Let

\[
Y = \sum_{i=1}^{n} X_i
\]

Find the following:
(a) the mean and variance of \( X_i \)
(b) the mean and variance of \( Y \)
(c) the mean and variance of \( Y/n \).
(d) \( E[Y^3] \).

2. Let \( Z \) be a standard normal random variable (standard means it has mean zero and variance one.) Let \( X = Z^2 \). Find the probability density function of \( X \). Hint: the standard trick is to first compute the cumulative distribution function \( F(t) = P(X \leq t) \). Then remember that the probability density function is obtained from \( f(x) = F'(x) \).

3. Let \( X_1 \) and \( X_2 \) be independent random variables, each of which has a Poisson distribution with \( E[X_i] = \lambda_i \).
(a) Use the moment generating function to compute \( E[X_1^3] \) and \( E[X_1^2 X_2^2] \).
(b) Use the moment generating function to show that \( X_1 + X_2 \) has a Poisson distribution.

4. (a) Derive the formula for the moment generating function for the normal distribution.
(b) Let \( X_1, X_2 \) be independent normal random variables with means \( \mu_1, \mu_2 \) and standard deviations \( \sigma_1, \sigma_2 \). Show that \( X_1 + X_2 \) is a normal random variable and find its mean and standard deviation.

5. I flip a fair coin 1000 times. \( X \) is the number of heads I get.
(a) Find the mean and variance of \( X \).
(b) Use the central limit theorem to find (approximately) the probabilities that \( X \geq 510, X \geq 550, X \geq 600 \).
(c) Find (approximately) \( E[X^4] \).