Math 466 - Homework 2

1. Problem 1 from chapter 2 in the book.

2. Problem 2 from chapter 2 in the book.

3. Problem 4 from chapter 2 in the book.

4. Problem 8 from chapter 2 in the book.

5. Recall that events A and B are independent if $P(A \cap B) = P(A)\,P(B)$. As we observed in class, if the random variables 1_A and 1_B are independent, then the events A and B are independent. Prove the converse: if the events A and B are independent, then the random variables 1_A and 1_B are independent. You must show that for any functions $g(x)$ and $h(x)$,

$$E[g(1_A)h(1_B)] = E[g(1_A)]\,E[h(1_B)] \tag{1}$$

6. Let X be a random variable with finite mean and variance. Prove that for all constants c,

$$E[(X-c)^2] \geq E[(X-\mu_X)^2] \tag{2}$$

7. Let $A_1, A_2, \cdots, A_n$ be independent events with the same probability p. We studied the problem of estimating the population proportion p by using the sample proportion. We saw that $f_n = \bar{X}_n$, i.e., the sample proportion is equal to the sample mean of the random variables $1_{A_1}, \cdots, 1_{A_n}$. This is an estimator for the population proportion, p. The variance of the random variable 1_{A_i} is $p(1-p)$, but p is unknown. Since $\bar{X}_n$ is hopefully close to p, a possible estimator for the variance $p(1-p)$ is $\bar{X}_n(1-\bar{X}_n)$. Show that the mean of this estimator is

$$E[\bar{X}_n(1-\bar{X}_n)] = p(1-p)\frac{n-1}{n} \tag{3}$$