

Math 566 - Homework 2

1. Problem 1 from chapter 2 in the book.
2. Problem 2 from chapter 2 in the book.
3. Problem 4 from chapter 2 in the book.
4. Problem 8 from chapter 2 in the book.
5. Recall that events A and B are independent if $P(A \cap B) = P(A)P(B)$. As we observed in class, if the random variables 1_A and 1_B are independent, then the events A and B are independent. Prove the converse: if the events A and B are independent, then the random variables 1_A and 1_B are independent. You must show that for any functions $g(x)$ and $h(x)$,

$$E[g(1_A)h(1_B)] = E[g(1_A)]E[h(1_B)] \quad (1)$$

6. (a) Let X be a random variable with finite mean and variance. Prove that for all constants c ,

$$E[(X - c)^2] \geq E[(X - \mu_X)^2] \quad (2)$$

- (b) Now assume also that X is continuous. The median of X is a number m such that $P(X \geq m) = 1/2$. (It need not be unique.) Prove that for all constants c ,

$$E[|X - c|] \geq E[|X - m|] \quad (3)$$

7. Let A_1, A_2, \dots, A_n be independent events with the same probability p . We studied the problem of estimating the population proportion p by using the sample proportion. We saw that $f_n = \bar{X}_n$, i.e., the sample proportion is equal to the sample mean of the random variables $1_{A_1}, \dots, 1_{A_n}$. This is an estimator for the population proportion, p . The variance of the random variable 1_{A_i} is $p(1 - p)$, but p is unknown. Since \bar{X}_n is hopefully close to p , a possible estimator for the variance $p(1 - p)$ is $\bar{X}_n(1 - \bar{X}_n)$. Compute the mean of this estimator, i.e., compute $E[\bar{X}_n(1 - \bar{X}_n)]$. Your answer should not be $p(1 - p)$ but should converge to $p(1 - p)$ as $n \rightarrow \infty$.