Math 466 - Homework 4

1. We want to test a hypothesis involving a population proportion. The unknown population proportion is p. The null hypothesis is $p = 1/2$ and the alternative hypothesis is $p > 1/2$. We want the level of the test to be 0.01, and we want the power of the test to be at least 0.9 when $p > 0.55$. Determine how large a sample we need and specify what the test is, i.e., when we accept the null hypothesis and when we reject it. Note that we worked out the power function for this setting of a one sided alternative in class. (It is also worked out in the book.) If you want to make the problem a little more challenging you can instead take the alternative hypothesis to be that $p \neq 1/2$.

2. Problem 1 in chapter 5 in the book.

3. Problem 2 in chapter 5 in the book.

4. Problem 3 in chapter 5 in the book. If you are using R you will find the following functions useful. pnorm() computes the c.d.f. of a normal distribution. sort() puts a sample in increasing order. (1:n-0.5)/n will create an array with the numbers $(i - 0.5)/n$ for $i = 1, 2, \cdots, n$.

5. Suppose X (the population) is a continuous random variable with probability density function

$$f(x) = \frac{\lambda}{2} \exp(-\lambda|x - \mu|), \quad -\infty < x < \infty \tag{1}$$

where λ and μ are unknown parameters. The mean of X is μ and the density is symmetric about $x = \mu$, so the median is also μ. We are given a random sample $X_1, X_2, \cdots, X_n$ with n large. Determine whether the sample mean or the sample median is a better estimator for μ. By "better" I mean "has smaller variance."

6. The random variable X is uniformly distributed on the interval $[0, \theta]$. (This is the population.) θ is an unknown parameter. We have a random sample X_1 of size 1. We want to use it to estimate the unknown parameter θ. Consider estimators of the form $T = cX_1$ where c is a constant.
(a) Find the value of c which makes this an unbiased estimator.
(b) Find the value of c which minimizes the mean square error. This is the risk when we take the loss function to be $(T - \theta)^2$.

If you want a more challenging version of this problem, see the 566 homework.