

3. Consider the exponential distribution \( f(x|\theta) = \theta e^{-\theta x} \) where \( \theta > 0 \). As always, we have a random independent sample \( X_1, X_2, X_3, \ldots, X_n \). The mean of this distribution is \( \mu = 1/\theta \).
   
   (a) Find the maximum likelihood estimators of the mean \( \mu \) and of \( \theta \).
   
   (b) By appealing to a theorem, show that for large \( n \), the MLE for \( \theta \) is approximately normal, with mean \( \theta \) and variance \( \theta^2/n \).

4. Consider the geometric density \( f(x|p) = p(1-p)^x \) where \( x = 0, 1, 2, \ldots \). We have a random independent sample \( X_1, X_2, X_3, \ldots, X_n \). Find the maximum likelihood estimator of the mean and of \( p \).

5. Consider the uniform distribution on \([0, \theta]\). We have a random sample \( X_1, X_2, \ldots, X_n \).
   
   (a) Find the maximum likelihood estimator of \( \theta \). Hint: don’t use derivatives. Just try to maximize the likelihood given \( X_1, \ldots, X_n \).
   
   (b) Find the MLE of the mean \( \mu = \theta/2 \).
   
   (c) (566 only) Now suppose that we have the uniform distribution on \([\theta_1, \theta_2]\) with both \( \theta_1 \) and \( \theta_2 \) unknown. Find the MLE’s of \( \theta_1 \) and \( \theta_2 \) and of the mean \( \mu = (\theta_1 + \theta_2)/2 \).