Math 466/566 - Homework 5

1. Book, chapter 7, problem 4.

2. Book, chapter 7, problem 5.

3. Consider the exponential distribution $f(x|\theta) = \theta\, e^{-\theta x}$ where $\theta > 0$. As always, we have a random independent sample $X_1, X_2, X_3, \cdots, X_n$. The mean of this distribution is $\mu = 1/\theta$.

(a) Find the maximum likelihood estimators of the mean μ and of θ.

(b) By appealing to a theorem, show that for large n, the MLE for θ is approximately normal, with mean θ and variance θ^2/n.

4. Consider the geometric density $f(x|p) = p(1-p)^x$ where $x = 0, 1, 2, \cdots$. We have a random independent sample $X_1, X_2, X_3, \cdots, X_n$. Find the maximum likelihood estimator of the mean and of p.

5. Consider the uniform distribution on $[0, \theta]$. We have a random sample $X_1, X_2, \cdots, X_n$.

(a) Find the maximum likelihood estimator of θ. Hint: don't use derivatives. Just try to maximize the likehood given $X_1, \cdots, X_n$.

(b) Find the MLE of the mean $\mu = \theta/2$.

(c) (**566 only**) Now suppose that we have the uniform distribution on $[\theta_1, \theta_2]$ with both θ_1 and θ_2 unknown. Find the MLE's of θ_1 and θ_2 and of the mean $\mu = (\theta_1 + \theta_2)/2$.

6. (**566 only**) Book, chapter 7, problem 6.