Math 466/566 - Homework 7

1. (a) The population has a gamma distribution with $\alpha = 1$ and β unknown. We want to test the null hypothesis $\beta = \beta_0$ against the alternative $\beta > \beta_0$ with a sample size of 100 and a significance level of 5%. Find the uniformly most powerful test.

(b) (566 only) Same question as part (a), except the population has a gamma distribution with $\beta = 1$ and α unknown.

2. Suppose that the population has a Poisson distribution. Find a sufficient statistic.

3. Suppose that the population has a uniform distribution on $[\theta_1, \theta_2]$ where both θ_1 and θ_2 are unknown parameters. Find a pair of sufficient statistics.

4. A physics laboratory has a radioactive substance whose decay time x has distribution

$$f(x|\theta) = \theta\, e^{-\theta x}$$

(The decay time x is always positive.) The parameter θ is unknown, but we know the substance is one of two kinds, so there are only two possible values for θ; call them θ_0 and θ_1. We have a random sample of n decay times, $x_1, x_2, \cdots, x_n$. We take a Bayesian perspective and assume (based on what we know about the contents of the lab) that the probability we have the material with $\theta = \theta_0$ is 9/10. We want to make a decision, based on the random sample, between $\theta = \theta_0$ and $\theta = \theta_1$. We assume that the loss is zero if we make a correct decision, and the loss is the same for the two possible types of errors. What is the test? Your answer should divide the set of possible samples $x_1, x_2, \cdots, x_n$ into two subsets, one where we decide $\theta = \theta_0$ and the other where we decide $\theta = \theta_1$.