Math 466/566 - Homework 8

1. Data set R.1 contains 10 pairs of points (x_i, y_i). Use this data for the following questions.

(a) With a significance level of 5%, test the hypothesis that the slope of the regression line for simple linear regression is 1.

(b) Find a 90% confidence interval for the vertical intercept, α of the regression line.

2. Let $\hat{\alpha}$, $\hat{\beta}$ be the estimators we found for simple linear regression.

(a) Find the mean and variance of the random variable $\hat{\alpha} + \hat{\beta}$. Caution: $\hat{\alpha}$ and $\hat{\beta}$ are not independent.

(b) The regression line is $\alpha + \beta x$. We want to test the null hypothesis that $(1, 1)$ is on the regression line against the alternative hypothesis that the line passes below $(1, 1)$ with significance level 5%. Define an appropriate statistic to use and specify the test, i.e., when we reject the null hypothesis. Your test should involve the data points (x_i, y_i), but not α, β or σ.

3. A patient's response y to a new drug is to be related to his or her response x to an old drug. We take the regression function to be quadratic, i.e., $\beta_0 + \beta_1 x + \beta_2 x^2$. So this is the mean of y. The data set R.2 contains 10 data points (x_i, y_i). Find the maximum likelihood estimators of $\beta_0, \beta_1, \beta_2, \sigma$. Find a 95% confidence interval for β_0, β_1 and β_2.

The following problem is optional. You can do it for fun, or you can do it in place of one of the above three.

4. Consider general linear regression with the usual parameters $\beta_1, \ldots, \beta_n$ and σ^2. Let $c_1, \cdots, c_n$ and d be constants. We wish to test the hypotheses:

$$H_0: \quad \sum_{j=1}^{k} c_j \beta_j = d, \qquad H_1: \quad \sum_{j=1}^{k} c_j \beta_j \neq d$$

(a) Show that $\sum_{j=1}^{k} c_j \hat{\beta}_j$ has a normal distribution and find its mean and variance.

(b) Let $c = (c_1, \cdots, c_n)$. Define a statistic by

$$T = \frac{\sum_{j=1}^{k} c_j \hat{\beta}_j - d}{\hat{\sigma}(c, (X^t X)^{-1} c)^{1/2}} \tag{1}$$

where $\hat{\sigma}$ is $S/\sqrt{n-k}$, the unbiased estimator for σ Show that if H_0 is true then T has student's t distribution with $n-k$ degrees of freedom.

(c) Specify the test for H_0 vs H_1 at a given significance level α.