

Here are known updates to the book ‘Function Field Arithmetic’. The major ones are (1) Progress towards generalization of soliton theory to general  $A$  situation: Anderson’s two-variable stark conjectures, in language of Tate’s thesis, with partial proofs, (2) Progress in transcendence theory because of Anderson, Brownawell, Papanikolas paper mentioned in the book leading to subsequent important works by Papanikolas, Jing Yu and Cheih-Yu Chang, (3) Development of Ihara power series, multizeta relations and related structures due to Anderson and Thakur, (4) Progress in understanding good extensions of  $t$ -motives (shtukas) and relations to L-values in important work by Vincent Lafforgue and Lenny Taelman, leading to Bloch-Kato, class no. formula type results and analogs of Class module etc., (5) Work of Pink, his students and Hartl etc. on developing hodge theory and adelic openness of Galois images type results.

I will be grateful to receive more updates.

1. Pa. 15, paragraph before last, product formula was generalized to number fields and given rigorous interpretation by V. S. Vladimirov (1996-2003). (Nov04)
2. Pa. 29, function field analog of Shafarevich conjecture, namely that the absolute Galois group of the maximal abelian extension of function field over a finite field is free profinite on countable many generators, had been proved by David Harbater in a recent preprint, based on another analog where ‘maximal abelian’ is replaced by ‘maximal cyclotomic’ meaning ‘maximal constant field extension’ version that he proved in 1995. (Mar. 2007).
3. Pa. 45, that  $p_d = D_d$  follows from comparison of coefficients of  $x^{q^d}$ , since  $e_d$  is monic. The argument given here proves recursion also independently, but not needed for just this equality.
4. Pa. 130-140, 4.13-4.15: The book ‘Integer valued polynomials’ by P. J. Cahen and J. L. Chabert (AMS 1997) is a general reference on related classical material. The recent thesis of David Adam of Universite de Picardie Jules Verne (2004) shows non-existence of simultaneous  $p$ -orderings in some function field situations giving analog of Melanie Wood’s JNT 2003 result. (May04)

5. Pa. 145, last para. The identity referred follows more directly by expanding the generating function in geometric series. (Sep 05)
6. Pa. 194 last para. Period interpretation of  $\zeta_d$  is achieved (Aug. 05)
7. Pa. 206-207, Poonen's results have been improved upon by Andreas Schweizer (Math. Z. 2003). (Nov04)
8. Pa. 229, Kochubei has written several papers on differential formalism and analysing Hypergeometric and other equations. In particular, he uses the remark on pa. 229 that  $\Delta_a = \Delta - [-a]$ , if  $a \in Z$  to have a variant definition of  $\Delta_a$  with  $a$  in function field and agreeing with this for  $[-a]$ , thus looks at hypergeometric function with finite characteristic parameters. Thus in the general situations, nice looking analogies eg. for contiguous relations become a little more complicated due to corresponding twists one has to introduce. (May 08).
9. Pa. 247, 7.4, Pink, Hartl, Gardeyn, Boeckle have proved several results on uniformizability, including Berkovich openness of uniformizable locus. (May 05)
10. Pa. 267, 8.1.3 (2) has been solved. (Aug. 05).
11. Pa. 280, 8.4, In Article 'Two variable refinement of the stark conjectures in function fields' to appear in Compositio, Anderson has given yet another beautiful approach to solitons, using the framework of Tate's thesis, and has given conjectural generalizations to any function field using very nice natural adelic approach. (May 05) Compositio 142 (2006), 563-615 (Nov 06) Progress towards proof is in Crelle (2007) and Annals d'Institut Fourier (2007) papers by Anderson. (May 08).
12. Pa. 314-315, Thanks to Alain Lasjaunias, who pointed out to me that de Mathan had proved improved version of Lemma 9.3.3 and using that in his thesis Lasjaunias had proved improved version of 9.3.4 working for all elements of the form discussed in 9.3.5 (taking care of the gap mentioned there). (Sep. 11)
13. Chapter 10, As mentioned above, there have been dramatic improvements in the results mentioned here, due to implications of ABP criterion mentioned in the book: Papanikolas (Inventiones (2008)) has

proved analog of Grothendieck conjecture on period and motivic relations in this setting and developed appropriate theory of difference Galois groups for Shtuka type equations. Using this, full algebraic independence results have been established for (i) logarithm values (Paper of Papanikolas above), (subsequently Pellarin has given another proof using approach of Denis who had earlier proved a weaker algebraic independence result) (ii) Zeta values (Chieh-Yu Chang, Jing Yu *Advances in Math* (2007)), (iii) Zeta values with varying  $q$ , geometric gamma values together with zeta values (Chang, Papanikolas, Yu preprint 2008), (iv) Zeta values together with Arithmetic gamma values (Chang, Papanikolas, Thakur, Yu, preprint 2008). The last result thus improves the result settling transcendence of arithmetic gamma monomials at fractions that was first proved by automata (mentioned in chapter 11), but not the  $v$ -adic such result or Mendes-France, Yao result proved by automata. (v) Chang and Papanikolas also have some partial independence results with Drinfeld modules of higher rank. (May 2008)

14. Pa. 345, the open questions mentioned in the third paragraph from the bottom have been settled in positive by Adamczewski, Bugeaud, Luca in *C. R. Acad. Sci. paris, Ser. I* 339 (2004), 11-14 and a preprint 'on the complexity of algebraic numbers' by Adamczewski, Bugeaud. (Nov. 04).
15. Pa. 355, Theorem 11.4.4: This theorem was curiosity with characteristic zero irrationality hypothesis (expected true) implying finite characteristic transcendence via automata. Sorosh Yazdani, *J. Th. des Nombres de Bordeaux* 13 (2001), 651-658 improves by removing the hypothesis of this theorem by using automata characterization (iii) of Theorem 11.1.3. (August 04)