The purpose of this file is to provide corrections and references to further developments regarding the material in the publication list. Any questions, comments, suggestions and corrections are most welcome. Especially, if you know any progress about the open questions mentioned in my papers, which is not mentioned in these updates, please let me know.

This file is unfortunately, quite outdated as i have not worked on it systematically for a few years. But I keep on adding pointers once in a while. Most updates and corrections have been incorporated in my book 'Function Field Arithmetic', and can be found on my homepage. But I still would add misprints/corrections here.

This file will be always under construction. Right now the corrections are in better shape than updates, which are incomplete and in the form of informal pointers to myself. Eventually I hope to make precise references.

The main basic references for the arithmetic of function fields, in the book form are (A) Springer Lecture notes 1231 by Gekeler on Drinfeld modular curves, (B) Proceedings 'Arithmetic of function fields', of conference in 91 at Ohio University, pub. by Walter de Gruyter and edited by David Goss, David Hayes and Michael Rosen, (C) 'Basic structures of function field Arithmetic' by David Goss, Ergibnesse published (1996) by Springer Verlag. (D) 'Function Field Arithmetic' by Dinesh thakur (2004), pub. by World Scientific. These contain extensive bibliographies and I plan to refer to them soon.

The updates covered in later papers are not usually mentioned.

The corrections to my old papers are usually given at the end of my more recent papers: we will also collect those below.

Papers have been identified by numerals referring to the publication list, as well as by catch words, journal and year.

1. (1) (Hengelhoef 86) (Updates) See ((I) Thesis 87).

2. (I) (Thesis 87) (Corrections): Most of the misprints are corrected in Gamma paper, but anyway:

   Pa. 14, 3rd displayed eqn. and also the one on the last line should be $(D_0 \cdots D_n)^{q-1} = D_{n+1}/([1] \cdots [n+1])$.

   Pa. 16, In the second line in the second claim, $g(0)$ should be replaced by $g(0)^{1/2}$.

   Pa. 20, In the second displayed formula, the last exponent should be q^d rather than $1 - q^d$.

Pa. 26, $M_r = []$ should be $M_r = -\lim[]$. Last line: $(-P_1)^{t_r}$ should be $(-1)^{h-1}(-P_1)^{t_r}$.

Pa. 27, thm. 3.1: $\Gamma_P(q^r/(1-q^r)) = ((-1)^{h-1}(-P_1)^{t_r})^{1/(q^h-1)}$.

Pa. 31, In theorem 3.5, $\tilde{P}$ should be replaced by $\tilde{\wp}$. Monicity condition before lemma 3.6 is only for $F_q[t]$.

Pa. 36, in the definition, need b of finite order. Also better to put sign condition on α in the hypothesis, ord_∞ stands for $\sum n_{\infty_i} v_{\infty_i}$. In the second line in the proof, the exponent is q^{hf-1} rather than $q^{hf}-1$.

Pa. 40, in the summation in the third paragraph, 'n monic' is missing, and in (**) on the last line we need $\Lambda_{m-1}^{(k)}$ rather than $\Lambda_m^{(k)}$.

Pa. 41, 3rd line last term should be $[k]^q/[1]$ rather than $[k]/[1]$.

(Updates) Gamma chapter has appeared in (V) (Gamma Annals 91), Gauss sum chapter has appeared, with different proofs and better formulation in (II) (Gauss Inven. 88). The Zeta chapter, which contained the first explicit transcendence and irrationality results on Carlitz zeta values, did not appear, as it was quickly improved by joint work with Greg Anderson (III) (Tensor, Annals 90) and by the work of Jing Yu (Annals 90, 96?). The preliminary results of this chapter were announced in (1) Hengelhoef conference paper (86).

Work of Cherif, de Mathan, Hellagouarch, Dammame, Allouche, Berthe, Denis: Detailed references will be provided later.

3. (II) (Gauss, Inv 88): (Updates) For A, other than $F_q[t]$, the theory is wildly different: See (VI) (Gauss JNT 91) and (IX) (Shtukas Invent. 93) and (X) (Gauss BLMS 93).

   Greg Anderson, Zhao, Feng and Chapman. Ref.?

4. (2) (Gross-Koblitz, Trento 89) (Updates) See (IX) (Shtukas, Invent. 93).

5. (II) (Tensor, Annals 90) (Updates) Applications to transcendence and generalizations to higher genus and class numbers. Jing Yu's papers in Annals 90, 96, Ohio volume, Anderson's papers in Duke, JNT, Dammame's thesis and Ohio paper, Dammame-Hellagouarch paper in JNT.

6. (IV) (Zeta measure JNT 90) Meaning and generalization to other A's still open. Carlitz claim: Javier Diaz-Vargas, Poonen, Sheats.

7. (3)(Analogies, BMC 90) (Corrections) On pa. 86, the first part of the last dispalyed equation should be $z/e(z) = 1 - \sum(z/\tilde{\pi}a)/(1-(z/\tilde{\pi}a))$.

   (Updates) Some more refined analogies in (5) (Iwasawa, CM 94).

8. (V) (Gamma, Annals 91) (Corrections): The last exponent $1-q^d$ in the first formula in 3.4, should be replaced by q^d. In 7.8, K-linearly should read Q-linearly. On pa. 50, one g is genus. On pa. 51, first displayed formula line 3, exponent of q is j-c: it should be mentioned that c is constant comming from Riemann-Rocah, and it is -1 for rational field. The reference [Y1] in 1.6 is wrong and is corrected in (4) (Gamma, Ohio 92) paper end.

   (Updates) Anderson, Sinha papers, (Gamma, Ohio 92), (XV) (Gamma, Annals 96), (IX) (Shtuka, Invent. 93) , (XVI) (Soliton). Bae-Yin-Yu preprint have some answers to questions at end. Brownawell-Papanikolas preprint generalizes Sinha.

9. (VI) (Gauss JNT 91) (Corrections) pa. 246, line -14, χ_{j-1} should be χ_{j+1}.

   (Updates) On pa. 244, it is shown that ρ_I determines ρ, if it is known to be sgn-normalized and it is stated that it is not known what happens if we drop the normalization. Lingsueh Shu provided me the following simple argument which shows that ρ_I for non trivial I does not, in general, determine ρ: Let A be with $\delta = 1$ and with class number more than one and with a place I of degree one. Let $\rho_I = c + F$, with $c \in H$. Let σ be a non-trivial element of the Galois group of H over K. Let μ be a $q-1$-th root of c/c^σ. Then clearly, ρ^σ and $\mu^{-1}\rho\mu$ are non-isomorphic, but both have I-th isogeny $c^\sigma + F$. (Since $(\mu^{-1}\rho\mu)_I = 1/(\text{leadingterm})(\mu^{-1}\rho_I\mu)$).

   Open questions on pa. 250: (1) and (3) are answered in positive in (IX) (Shtukas, Invent 93) and different proofs are given together with higher class number results and relation of factorization to theta divisor. For (4), see Anderson and Zhao papers in JNT. For more details on assertions in Cyclotomic theory summary, see Hayes paper in Ohio proceedings.

10. (VII) (Continued JNT 92) (Update) See (XIII)(Continued JNT 96) and (XVII)(Patterns). Allouche Toeplitz connection. Van Hamme student Ann Verdoot, Annales Math. Blaise Pascal, 1, 1994, 71-84 Continued fractions for finite sums.

    (Correction) On pa. 153, there is a sign misprint in displayed formulas for x_{n+1} and $\overline{x}_{n+1}$: The exponents $q^n(q-2)$ should be $-q^n(q-2)$. (Thanks to Diana Mecum for pointing this out).

11. (VIII) (Zeta, IMRN 92) (Corrections): In the published version, by mistake of the journal, the following abstract was dropped: In this paper, arithmetically interesting quantities such as factorial, gamma functions, binomial coefficients, zeta functions, in the context of function fields, are related to quantities connected with Drinfeld modules. This is then applied to obtain the algebraicity of Drinfeld exponentials of some special zeta values. The analogues, due to Jing yu, of Hermite-Lindemann and Gelfand-Schneider theorems about the transcendence properties of the exponential then imply transcendence of such values and their ratios with the periods.

    pa. 196, line 2, F_4 should be F_q.

    Pa. 188, line -7: $g-1$ should be $2g-1$.

    (Updates) Conjecture E is proved by Anderson, Hypothesis (almost). Refer to Anderson's two papers.

12. (4) (Gamma:Ohio 92) (Corrections) Last paragraph of (4) on pa. 85 is garbled: The interpolation we have defined earlier works fine and agrees with what is said there under the correct hypothesis that denominator of z divides $v-1$ missing there. There is no need of redefinition.

    The second to last statement in (5) pa. 85 should be deleted, as we do not yet know v-adic interpolations.

    (Updates) Anderson, Sinha papers, (Gamma, trans, Annals), (IX) (Shtuka, Invent 93) and (XVI) (Soliton). Mention Local gamma, gamma ideal, shu's paper. In Crelle 1997 paper (and preprints) Manjul Bhargava has given a very nice general recipe for factorials which specializes to usual notion for Z and Carlitz factorial for $F_q[t]$, but is different in general. (Remark is put in soliton paper). There does not seem to be simultaneous P-ordering for higher genus and if you do it for each prime, one ends up with Sinnott formula (Goss gamma ideal): whose local nature is explained in Shu (JNT).

13. (IX) (Shtukas, Invent 93) (Corrections) 5.1 to 5.4: The hypothesis $\delta = 1$ is missing.

    Pa. 559, line 3, the first $\overline{X}$ should be X.

    Pa. 560, line 2 and 3 from bottom, minus sign is missing between '$W = V$' and '$(\xi) + (\eta)$', so it should be $W = V - (\xi) + (\eta)$.

14. (X) (Gauss, BLMS 93)

15. (5) (Iwasawa, CM 94) Shu, Anderson and Mazur on class groups and Vandiver.

16. (XI) (Hypergeometric, FFA 95) (XVIII) (hyp. II) has updates. Kochubei. On pa. 229, $log(1 - z)$ should be $-z$ times $F(1, 1, 2, z)$. Analog of this corresponds to $\sum z^{q^n}/[n]$ rather.

17. (XII) (Zeta, Compositio 95) (Corrections) Pa. 232 First paragraph, last line: r_1 in the exponent should be $r_1 s$.

    Pa. 237: In formula for S_k, minus sign between $\mathcal{J}^{-1}$ and $\{0\}$ is missing. In third paragraph, one closing bracket is missing after $(j_2 + r(i)$.

    Pa. 239: Last but one paragraph: Reference to Theorem 2, should be to Theorem 1.

    Pa. 240: Remarks (ii): l(6infty) should be 2 and not 3, where it is mentioned, but in the next example of voloch, it should be 3.

    Pa. 245: Last paragraph: 'The results of Goss-Sinnott mentioned above': Somehow I forgot to mention these results: For K of class number one, for which there is exceptional vanishing, they imply non-vanishing of class group components of the $\wp$-th cyclotomic extensions of K, for any $\wp$.

    (Updates) A theorem and several examples of extra vanishing for higher class numbers are obtained in my student's thesis: 'On zeros of characteristic p zeta functions', Javier Diaz-Vargas, U. of Arizona, May 96. On the overall subject of the paper, see Goss book, Wan, Taguchi-Wan, Diaz-Vargas, Sheats for latest developments.

18. (XIII) (Continued JNT 96) (Correction) The formula for w_i on page 253 (also quoted in Thm. I in patterns paper) has to be multiplied by an appropriate element of F_q^* by taking into account the sign of the denominator of the initial convergent.

    On pa. 254, there is a sign mistake in definition of r_2: The numerator $-(bg+b'D)$ should be $(bg-b'D)$. (Thanks to Diana Mecum for pointing this out).

    (Update) (XVII) (Patterns, JNT 97) solves the main open question. A version of the Folding Lemma seems to have appeared even before the reference [S1] I gave, in Acta Arith. 23 (1973), 207-215 in an article by Mendes France.

19. (XIV) (q transc, JNT 96) (Correction) Ref. to Voloch should be Vol. 58, no. 1, 55-59 rather than vol. 57 no. 2.

    (Update) See (6) (Automata) paper and (XVIII).

20. (XV) (Gamma trans, Annals) (Corrections) In theorem 2, the hypothesis that 'α_l are not all zero' is missing.

21. (6) (Automata) paper.

    (Updates) Allouche, Mendes France, Yao.

22. (XVII) (Patterns JNT97) (Correction) See XIII. (Updates) $\theta \neq 1$ still open for $q = 2$. (Work in Progress: My student Aaron Ekstrom at Arizona has formulated some conjectures in this case.)

23. (XVIII) (Automata, PAMS).

24. (XXIX) (Hyp II) (Update) Kochubei's work.

25. (XX) (Computational classification IMRN)

    (Update) Dale Brownawell pointed out to me that Mahler refined his classification mentioned in the paper after 30 years in the reference Acta Arith. XVIII (1971): On the order function of a transcendental number, to get many classes, but it is hard to show that elements exist in his classes.

26. (XXI) (Dioph approx) (update) Abhyankar conference paper improves and updates this.

27. (XXII) (Log-alg.)

28. (XXIII) (Exponents and Deformation) (Correction) Page 597, note added in the proof: γ there is misprint for r.

29. (7) TIFR paper: m should be $m_\wp$. Kazdan-Flicker seems to have done Langlands with special condition at ∞, at least after later work. Lafforgue completes anyway.

30. (ii) In the cyclotomic fields book, pa. 328 paragraph last but one, $p = 2$ is misprint for $a_p = 2$.

    Pa. 166, para.4 line 6, modulo 3, when p=3 should be modulo 9 when p=3.

    Pa. 37, 3ed paragraph, 4th line: 'b^2' should be dropped.

    Pa. 39, 3rd para., 5th line, 'cube of an ideal' should better be 'cube of a principal ideal'.

31. (9) Diophantine proceedings paperpa. 762, in Riccati equation $a\alpha 2$ is misprint for $a\alpha^2$.

32. Elliptic survey paper: (Corrections) HS reference missing (pa. 231) is Hindry-Silvermann, Inventiones 93, 1988, 419-450. Pa. 231, 4th para. reference [Ab, Y] should be [Ab, Ya].

    (updates) Here is a nice argument/email (March 25, 03) from Bjorn Poonen answering the question (pa. 232) of Fpbar gonality of modular curves raised in the survey: THEOREM: There exists a function p(n) such that for any separable extension of fields L/k and any (smooth, projective, geometrically integral) curve X over k with a k-point, the gonalities $G_k(X)$ and $G_L(X)$ of X over k and L respectively satisfy $G_k(X) <= p(G_L(X))$.

    PROOF: It suffices to consider the case L=ksep. I will use the notation L(X) to denote the function field of the base extension X_L. Fix f in L(X) of degree $d := G_L(X)$. By descent theory, the subfield of L(X) spanned by the Galois conjugates of f is L(Y) for some k-curve Y, and we have a dominant morphism $h : X - - > Y$ such that f = f' h, for some $f' : Y_L - - > P^1$. Since $[L(Y) : L(P^1)] <= [L(X) : L(P^1)] = d$, at most d conjugates of f' are needed to generate L(Y) over $L(P^1)$ (in fact this can be improved to $log_2 d$, by looking at how the degree of the generated field grows with the addition of each new conjugate). Let $f'_1, ..., f'_e$ be these conjugates, so $e <= d$. Then $f'_1, ..., f'_e$ map Y_L birationally to its image curve in $(P^1)^e$ of multidegree at most (d,d,...,d), so the genus g of Y_L is bounded in terms of d. Also, $G_k(Y) <= g + 1$, by Riemann-Roch applied to (g+1)P, where P is in Y(k). Finally, the degree of X–¿Y divides d, so $G_k(X)$ also is bounded in terms of d alone.

As we discussed, this can be used to bound the Fpbar-gonality of modular curves from below, given a lower bound on the Fp-gonality.

I asked Matt Baker about this and he wrote

"The general method of using descent theory together with birational maps to bound gonality over separable field extensions appears in a few places: in my thesis, in a paper of Silverman and Harris, and in an unpublished paper by Nguyen and Saito (paper 11 at http://www.math.s.kobe-u.ac.jp/HOME/mhsaito/)."

although he did not know an explicit reference mentioning the application to modular curves over Fpbar. Best regards, Bjorn

Refer to Poonen, Sheats, Denis, Thiery, Kapranov, Berthe, de Mathan, Koskas etc. salon, Buchi, Villmeyer, Allouche, Bae-Yin-Yu, Brownawell-Papanikolas.

Update Preprints references to reprints.