Modeling With Right Triangle Trigonometry

1. **The Gutter Problem:** A rain gutter is to be constructed by taking an aluminum sheet 12 inches wide, marking off 4 inches from each edge, and bending up the side at an angle \( \theta \).

![Diagram of the gutter problem]

**Question:** What angle should we make \( \theta \), in order to maximize the cross sectional area of the gutter? (This bend will allow the most water to flow through the gutter.)

What is the cross sectional area \( A \) of the opening of the rain gutter if the angle \( \theta \) is 25°?

Hint: Divide the cross sectional area into sections and find the missing (and important) parts of each section.

Area = ________________

Write a function \( A(\theta) \) for the cross sectional area of the opening of the rain gutter for any angle \( \theta \). Use algebra to simplify the expression.

\[
A(\theta) = \frac{4 \tan(\theta)}{2} = 2 \tan(\theta)
\]

With your calculator, sketch a graph of the area function \( A(\theta) \). (Again, think about DEGREES vs. RADIANS.) Think about the values of \( \theta \) that make sense to the problem.

Using your graph, find the angle \( \theta \) that maximizes the area of the opening. What is the maximum cross sectional area?

The maximum area occurs when \( \theta \) is ________. The maximum area is _________________ in\(^2\).

Using your graph, find the angle \( \theta \) that would cause the area of the opening to be 15 in\(^2\). \n
\( \theta = \) ________________
2. **Isosceles Triangle Problem:** The two equal sides of an **isosceles triangle** have a length of 10 cm, as shown below.

![Isosceles Triangle Diagram](image)

a. Let the **base angles** of the triangle measure 20 degrees.
   
   i. Calculate the **perimeter** of the triangle.

   ii. Calculate the **area** of the triangle.

b. If the **base angles** of the triangle measure \( \theta \) degrees, determine a function \( P(\theta) \) for the **perimeter** of the isosceles triangle, and a function \( A(\theta) \) for the **area** of the isosceles triangle.

\[
P(\theta) = \text{___________________________} \quad A(\theta) = \text{___________________________}
\]

(c. With your calculator, graph the function \( P(\theta) \), and use your graph to determine the angle \( \theta \) that will make the perimeter 25 cm.

   \[ \theta = \text{__________} \]

d. With your calculator, graph the function \( A(\theta) \), and use your graph to determine the angle \( \theta \) that will
   
i. make the area of the triangle 30 cm\(^2\).
   \[ \theta = \text{__________} \]

   ii. maximize the area of the triangle.
   \[ \theta = \text{__________} \]
3. **The Cone Problem**: Shown to the right is a cone with a slant height of 15 cm. Let's explore the relationship between the volume of the cone and the angle $\theta$.

Remember, for a cone, the Volume $= \frac{1}{3} \cdot \pi \cdot \text{radius}^2 \cdot \text{height}$.

**Question**: What angle should we make $\theta$ in order to maximize the volume of the cone?

Let's take a small angle for $\theta$, for example 5 degrees. Since the volume of the cone is dependent on the radius and the height of the cone, we need to calculate the radius and the height if the angle $\theta$ is 5 degrees.

Write a trig equation that you could use to determine the radius of the cone. Solve the equation to find the radius.

If $\theta = 5$ degrees, $r =$ ____________.

Write a trig equation that you could use to find the height of the cone. Solve the equation to find the height.

If $\theta = 5$ degrees, $h =$ ________________

What is the volume of the cone if the angle $\theta$ is 5 degrees?

Volume $= \text{_______________________________}$

To determine the “exact” value of $\theta$ that maximizes the volume of the cone, we need to write a function for the volume in terms of the angle $\theta$. Use trig to write the radius and the height of the cone in terms of $\theta$.

$\text{radius} = \text{________________________} \quad \text{height} = \text{________________________}$

Using these two equations, write an equation for the volume of the cone in terms of $\theta$.

$\text{Volume} = \text{_______________________________}$

Graph this function on your calculator. Think about the window you need!

Now, use your grapher to complete the following:

1. The maximum volume for a cone with slant height of 15 cm is _____________ and it occurs when the:
   \[ \text{angle } \theta \quad \text{is } \text{___________}, \text{ the radius is } \text{___________}, \text{ and the height is } \text{___________}. \]

2. If the volume of the cone is 300 cm$^3$, what would the angle $\theta$ be? _________________