There were two purposes to this research. One was to continue the whip project that was begun in the Fall, and the other was to develop experiments to be used in a first year graduate applied math class, taught by Dr. Tabor. Because Dr. Goriely, the supervisor of the whip experiment was in Europe all summer, nothing was done toward that, and the focus of the research was for Dr. Tabor’s class. The objective was to find three or four experiments, preferably applicable to high-speed photography, which contained some sophisticated applied mathematics. An enormous list of possibilities was researched, eventually narrowing the search down to three definite possibilities. Following are details of all experiments considered.

**Experiments:**

*Milk Drop:*

In this experiment, an attempt was made to reproduce Edgerton’s famous milk drop experiment (http://web.mit.edu/vrtour/movies/n2_edgerton_mv.mov). This experiment was set up by placing a Petri dish 10 cm above the table. Just enough milk was placed into the Petri dish to cover the bottom. A syringe containing milk was secured into place, and allowed to drop into the Petri dish. The height of the syringe above the Petri dish can be varied for varying results.
Shattering Glass:

One experiment that had the lab rather excited was the possibility of examining the shattering of a glass plate. There was surprisingly little material available on the subject, and with some extremely slow motion videos of glass being shattered, it was thought that a considerable amount of information could be learned about the way cracks propagate through material. Upon performing the experiment, however, the shattering of glass takes place far too quickly even for the laboratory’s sophisticated equipment.

Fig. 2. a-l. These non-consecutive images were taken at a frame rate of 2000 frames per second, and a shutter speed of 1/4000 s. The sequence shown here spans 1/50 s.

Fig. 3. A drop of colored water falling into clear water. This was done to demonstrate how the drop disperses into the water.
In capturing the impact of a droplet of water on a surface of water, it is often possible to observe with high speed photography the droplet bouncing on the surface. There are many variables which contribute to whether the drop will bounce or not, and if so, whether the high speed camera will be able to capture a quality image or not.

The setup for making the drop fall into the water includes one beaker of water, a reservoir of water, from which the drops come, and a syringe whose tip forms the drops. The syringe is connected to the reservoir by way of small rubber tubing which includes a flow control valve. The reservoir also has an on and off valve. In this way, the flow rate can be left stationary, while the flow is turned on and off. The glass beaker that was used had no markings on it, so that there would be no reflection of the writing in the image. To further reduce glare and reflection, a coffee can, painted black on the inside to reduce glare, was placed on the outside of the beaker. The beaker had a small hole drilled into it at the brim so that the beaker could be completely

![Fig. 4](image1.png)

Two consecutive images of glass being shattered: This is a thin plate of glass being shattered by a stainless steel marble. This video was shot at 16,000 frames per second, which means it takes less than 6.25*10^-5 seconds for the cracks to propagate through the plate.

![Fig. 5](image2.png)

The initial setup. Shown is the syringe at the level of the first beaker. The flow control valve can be seen halfway up the tubing.

![Fig. 6](image3.png)

The fiber optic light setup. Shown is the beaker full of water, and the two fiber optic arms which provided the light. The beaker is surrounded with a coffee can that is painted black.
filled and the water level would remain constant. When the syringe is placed near the water surface, and the reservoir valve is turned on, the flow control can be adjusted to give an appropriate drip rate.

It is not trivial getting the camera and lights set up in such a way that the image is easily visible. The configuration that seemed to work best in this series of experiments was to place the camera approximately 28 cm from the surface elevated at an angle about 30° above the table. For lighting, one dynalite 150 fiber optic power supply was used (A.G. Heinze Incorporated Model DL-150). The two fiber optic arms were then placed approximately 21 mm from the tip at an angle of about 20° from vertical. There was also one 1000 watt spotlight placed about 27 cm from the tip slightly higher than the level of the water. The camera had a number 0 and a number 2 close up lens, with 0 being closest to the camera. The aperture was on f4. With these settings, the syringe tip showed up at the top of the image, and its reflection showed up at the bottom.

The variables taken into account when trying to get the drop to bounce were amount of water in the reservoir, size of needle, and height of the syringe above the water surface. The two that ultimately seemed to be critical were the height of the syringe above the surface and surprisingly, the flow rate. The size of the needle may have slightly affected the frequency and conditions under which a bounce was observed, but did not seem to be critical. The amount of water in the reservoir is completely arbitrary.

To get a drop to bounce, the height above the surface of the water at which the drop is released must be just right. The way this height was measured in this experiment was by clamping the syringe to a vertical bar, and sliding the clamp up and down until the tip of the needle just touches the surface of the water. At this point, the clamp was tightened, and another clamp was tightened in place directly beneath it. Then by leaving the bottom clamp in place, the top clamp could be loosened and measured relative to the bottom one. The distance of the gap between the two clamps would equal the distance from the tip of the needle to the surface of the water. In this experiment, a stack of cover slides exactly one millimeter thick were used to measure this gap.

In order to see a bounce, often it was necessary to find the right height through trial and error. It is immediately obvious when the tip is too close to the surface because the drop seems to dissipate into the water before it is released from the syringe. In other words, the drop touches the surface of the water and the syringe at the same time. On the other hand, if the syringe is too far from the surface of the water, the drop seems to immediately penetrate the surface upon impact.

![Fig. 7: A drop that was released too close to the surface. This drop needs to be released slightly higher, so that it has time to form before it makes contact with the surface.](image-url)
Pinch Off:

In this experiment, drops were observed at distances ranging from 5mm above the surface to 12mm above the surface. Once a good distance is found somewhere between these two extremes, the drop still may or may not bounce, however, by modifying the drip rate with the flow control valve, it is a simple matter of experimentation to find a rate at which drops will bounce. It is suspected that this relates to the faucet drip problem and that at different flow rates, the water drop takes on different physical properties. This is supported by the fact that even at a constant drop rate, the bouncing will occur for a few drops, then it will stop. It is also proposed that the disturbances in the water surface affect whether or not the drop will bounce. It seems that when the drop rate is too slow, the drop doesn’t bounce, but when it is too high, it also doesn’t bounce, so it is possible, that there is a requirement for some disturbance in the water surface, but not too much.

The faucet drip experiment is a very famous problem in applied mathematics. To study this behavior, it is advantageous to study one fluid dropping through another fluid with similar specific gravity. The reason for this is that the effect is slowed down to the point that the phenomenon can more easily be observed. The purpose of this experiment was to determine a good way to photograph the dripping of one fluid into a slightly less dense fluid.

The experiment was performed by filling a small pipette tip with one fluid, and suspending it over a beaker containing another fluid. The pipette tip had an outer diameter of 1.4 mm. It was set at such a height that the tip was under the surface by three to seven

![Fig. 8: A drop that was released too far from the surface. This drop immediately breaks the water surface and penetrates.](image)

![Fig. 9: The setup. Shown is the camera, about 15 cm from the syringe tip, which is under the surface of the oil in the beaker.](image)
millimeters. The measurement of how far the tip was under water was not recorded, or even noted, because it was assumed that it was not important for the purposes of these preliminary experiments.

The setup was developed under the assumption that a silhouette of the falling drop would provide the best contrast. There was a white background behind the beaker, which was illuminated by two 1000 watt spotlights. In an attempt to get no light on the beaker, it was surrounded by a black tunnel. The camera had close up lenses of levels 0, 1, and 2, and was placed about 15 centimeters from the tip. The tip was held in place with a metal plate that had a hole in its center. The small pipette was held by a clamp, which rested on the metal plate. With this setup, the beaker could be filled with 200 milliliters of one fluid, with the plate and pipette resting on the beaker, and the other fluid could be inserted into the pipette with a syringe.

It took some experimenting to determine which two liquids would work best in this experiment. At first, corn syrup was dropped into vegetable oil. The pinch off phenomenon was observed, however the contrast was not great. Whenever both liquids were water soluble, it seemed that instead of pinch off, there was just a steady stream of liquid falling straight to the bottom of the beaker. It was attempted to drop motor oil and many different plant oils into water, however, they were all too light, and floated to the top. Motor oil was also too light to be dropped into vegetable oil, and furthermore the two liquids mixed. To prevent this mixing, it was
decided to use immiscible fluids. Glycerin and vegetable are immiscible, since glycerin is water soluble and oil is not. When colored glycerin was dropped into vegetable oil, the pinch off was observed. For the remaining experiments, red or blue glycerin was used. The red glycerin was not as visible as the blue, however, and it was abandoned.

With the appropriate liquids, it was trivial to observe a pinch off. The pipette was filled with glycerin, and allowed to freely flow. At first, when the pipette was still full (about 1.5 cc), the pinch off occurred near the bottom of the beaker. As the glycerin ran through the pipette, however, the pinch off occurred closer to the pipette tip. For these preliminary experiments, the amount of glycerin in the pipette was not measured, but the image was taken when the pinch off was within the camera view. The camera was set to frame rates of 250 to 2000 frames per second. The optimal setting was to have the camera at 2000 frames per second, with a shutter speed of $1/2000$ s. The best aperture setting was $f8$.

**Water Jet:**

The purpose of this experiment was to capture a jet of fluid penetrating the surface of another fluid with a high speed camera. In order to do this, one colored fluid was needed, and one transparent fluid. For simplicity, dyed water was dropped into transparent distilled water.

The setup of the experiment was simple. A syringe filled with the colored water was positioned above a beaker of transparent water with its tip placed approximately 2 to 3 mm above the surface. Two 1000 watt lights were used, both shining on a white backdrop behind the beaker from a distance of about 35 cm. The water was simply expelled from the syringe by the stopper, and the image captured.

The camera had three close-up lenses, numbered 0, 1, and 2. The camera was set at an aperture of $f8$ and was placed about 15 cm from the syringe tip. Images were taken at 1000 or 2000 frames per second. There didn’t seem to be any detail missing when the slower frame rate was used, and since it gave a bigger viewing area, this seems to be a more appropriate setting. The shutter speed was set to 2000 frames per second for all frames.

The water jet seemed to show up well, though in the future, a darker dye may be used for better contrast. After several water jet images were shot, it was noticed that when the colored liquid dripped into the water, a toroidal vortex was formed. Consequently, the remainder of the experiment was dedicated to capturing images of these vortex rings.

![Fig. 12: A jet of colored water penetrating a surface of water. This is a steady stream of the red water, which shows the jet going from laminar to turbulent.](image)
The rings were captured on high speed images, however, it is sometimes difficult to determine that they are vortices. As a result, the idea was had to place a small mirror at a 45 degree angle at the bottom of the beaker, so a front view and a bottom view could simultaneously be seen. The future of this experiment will be to develop a method of lighting this setup, so good images can be seen.

**Fig. 13:** Two toroidal vortices. These two vortices were formed by dropping a small amount of red water into a beaker of water. Soon after, the two vortices leapfrog and then become one vortex.
**Fog Rings:**

When studying fluid dynamics, there is much knowledge to be gained by studying the structures known as toroidal vortices. One example of such vortices is the fog rings generated by a Zero Launcher fog ring generator. The purpose for this experiment is to determine the best way to capture such images with a high speed camera, for later analysis.

One issue that needed to be confronted was the fact that there existed air currents through the room, which affected the fog ring, and added unnecessary variables to the experiment. To reduce this effect, an aquarium was placed upside down on the table with the fog ring inside. The aquarium was offset about ten centimeters for access. A black layer of material was placed on the inside of the aquarium along the side behind the fog ring. This served two purposes, one was to provide a black backdrop to contrast the rings, and another was to eliminate the reflection of the rings against the back of the aquarium. For better lighting, a mirror was also placed on the table, face up. It was not clear that this made a difference. For best results, the fog ring generator should be taped to the table, so that it does not move while the experimenter is attempting to generate a ring.

The camera was placed about 1 meter from the rings, with the center of the lens about 24 cm from the table top. Most images were taken using an aperture setting of f4 or f5.6. The images were taken at 1000 frames per second, which yielded an image size of 1280 x 512 pixels. A higher frame rate did not seem to be necessary, unless extremely accurate measurements were required. If a higher frame rate were used, however, the view would be smaller. The shutter speed was set at 1/2000 second. The images seemed to be sufficiently crisp at this shutter speed.

There were two light schemes used, both involving two 1000 watt spotlights. Each scheme had its own advantages and disadvantages. In both cases, the lights were level with the orifice from which the ring emerges. The first setup had both lights on the side of the aquarium facing the camera at 45 degree angles to the face of the aquarium. They were each aimed toward the opposite rear corner of the aquarium. The distance from the light to the axis along which the ring travels was approximately 40 cm. The advantage to this setup was that all fog showed up, however, there was less distinction between the thicker

![Fig.14. The first lighting setup. This setup causes more fog to show up in the images.](image)

![Fig.15. The second lighting setup. This setup emphasizes the denser fog.](image)
fog and the thinner fog. Furthermore, as the ring reached the end of its path, the lighting became increasingly poorer. The second setup involved moving the second light to the axis along which the ring travels. The light was placed about 75 cm from the fog ring generator, shining directly toward it. This setup showed thick fog well, and didn’t show thinner fog, so in some cases, the structure of the ring was more easily seen. However, the trail left by the ring did not show up. This setup solved the problem of decreasing light intensity, but with this setup, the ring became increasingly bright to the point of overexposure.

![Fig. 16. A smoke ring generated by the Zero Launcher. In this image, the first lighting scheme was used, and therefore nearly all fog was visible, including the trail left by the smoke ring.](image1)

![Fig. 17. An image created using the second lighting setup. This time the trail is not as visible, however the structure of the ring can be more easily seen, because less dense fog is not visible.](image2)

The process for capturing an end view of the fog ring involved an entirely different setup. The camera lens was about 1.05 meters from the zero launcher, and the two faced each other. The camera was aimed with the hole from the launcher near the top of its view. Once again, the aquarium was used to prevent drafts. A cardboard shield was placed over the ring generator to reduce background fog. The two lights were stationed about 34 cm from the center axis, immediately next to each other, and parallel. The center of the lights was about 99 cm from the ground. To improve lighting, a mirror was placed inside the aquarium opposite the lights. Each light contains a knob that diffuses the light. In the images taken, this knob was turned halfway. The aperture of the
lens was set to f8, and the images were taken at 250 frames per second, with a shutter speed of 1/250 second. The resolution for the images was 640 x 512.

Upon observing the fog rings from an end view, it was noticed that they contained instabilities around the outside circumference. It was suspected that the number of these instabilities might be correlated to the velocity of the ring, however a measure of velocity was not easy to come by, due to the view. Therefore, to get an estimate of the ring’s velocity, the distance from the shield to the end of the aquarium was measured, and divided by the time it took from when the ring was first lit up until it appeared to hit the glass at the end. From the four rings that were analyzed in this manner, it seemed that the faster the ring moves, the more instabilities it contained.
The next step in this project was to attempt to develop an accurate method of determining the exact position of the ring along the axis of the camera and fog generator. The first idea was to shine a laser pointer into the aquarium, so that when the ring passed by it, a more intense line was seen. Upon trying this, however, it became clear that the spotlights used to light the ring were much too intense in comparison with the laser pointer, and the laser pointer did not appear on the recorded image. It was then decided that perhaps a light curtain could be used to light the aquarium along a plane perpendicular to the camera/generator axis. Metal sheets were placed on the side of the aquarium, with a gap between each one 2mm wide. The 1000 watt spotlights were then shined through these gaps to provide the light curtain. Upon observing the resulting image, the ring did flash as it passed through these curtains, however, the general lighting was too poor to give a good image. No general lighting plan was found that was intense enough to suitably light the whole aquarium. Although development of the lighting was halted, some additional work could provide better results.

It became of interest to observe the interactions between two fog rings. To do this, the setup was left as it was for the end view, but another fog ring generator was

\[\text{Table. 1. The numerical data on the four fog rings that were analyzed. Fig. 20. A graph of the velocity vs. the number of “instabilities”.}\]

<table>
<thead>
<tr>
<th>Image</th>
<th>Start Frame</th>
<th>End Frame</th>
<th>Difference</th>
<th>Time (s)</th>
<th>Velocity (m/s)</th>
<th>Instabilities</th>
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</tr>
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</table>
placed next to the original one. They were on either side of the center axis, and pointed inward at an angle of around ten degrees. They were spaced far enough apart that a small piece of black cardboard was used in the center for a background. The rings were shot simultaneously, and the image captured. It was not trivial to capture two rings that collided. In fact, it is very difficult to get two dense rings, moving at the same speed to cross the same point at the same time. The best way was to fire them repeatedly, until two collided, and the image was then shot. Future work would most likely include a search for a better method of firing the rings, so that it is easier to get two to collide. Also, it may be of interest to capture a head on collision of two rings.

Fig. 21. The setup for the colliding fog rings.
**Conclusion:**

Upon reviewing the results, it was decided that the three best experiments for the purpose of the class were the fog rings, the pinchoff, and the milk drop experiment. Future research would include a search for the applied mathematics involved in such experiments. To perform an experiment with high speed photography, it was observed that lighting is an enormous factor. In many cases, it took a good part of a day to find a lighting setup that worked, and showed the object as desired. In the future, it is recommended that if an experiment is to be performed, allow ample time to find a lighting scheme that is satisfactory.

*Fig. 22. a-d. Four non consecutive images of two fog rings colliding.*