Progress Report

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Introduction

Sand fields develop different patterns, such as sandbars, sand ridges, sand ripples, sand dunes, and river dunes. These vary in shape from crescent to ridge-like patterns. They are formed in areas such as deserts, lakes and seashores, where winds or fluid flows are strong and tend to blow from one direction or oscillate. There is a great deal of order in these patterns: perhaps a simple model could be built that captures them. Here we use cellular automata to recreate them and to understand their dynamics.

The objective of this research is to model and develop patterns of the movement of sand; namely to study sand dynamics. Sand dynamics is the study of the behavior of sand when there is a force (such as wind, or fluid) acting on it. We present a simple model of a box of sand (see figure 1.a and b [1]), and discuss its dynamics in terms of two mechanisms: advection and avalanching respectively.

Cellular automata are the general name assigned to models for the collective behavior of large numbers of individuals. These individual members "agents" are assigned single dynamical rules. ([2], [3]) Cellular automata (CA) processes and related iterative processes have been used to understand the underlying structure of such complex system as clouds, riverbeds, and lava flows. In this study, we apply CA models to recreate the patterns of sand as seen in nature.

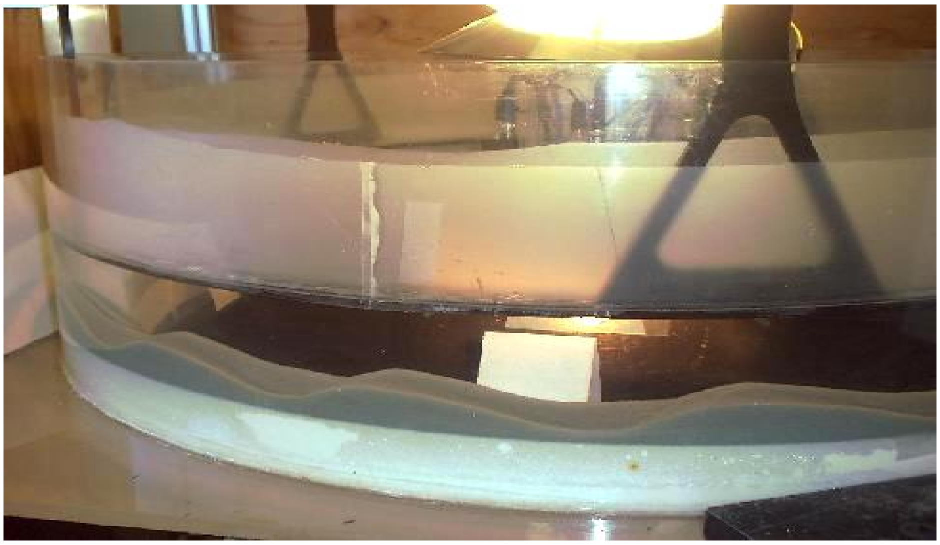


Figure 1. Sand Box in Laboratory. [1]

I. Prior Results

We previously worked on a one-dimensional model of sand dynamics, based on two fundamental mechanisms: advection and avalanching. The answers to explore how these competing mechanisms could produce sand patterns.

Numerical Model

In order to investigate whether patterns of sand fields have an underlying model, we establish a numerical model with a cellular automata computer code and use this code to simulate the evolution of variables, such as the height, inter-bar spacing, and speed of bars. We illustrate the box of sand in Figure 2. It is a two-dimensional box as below:

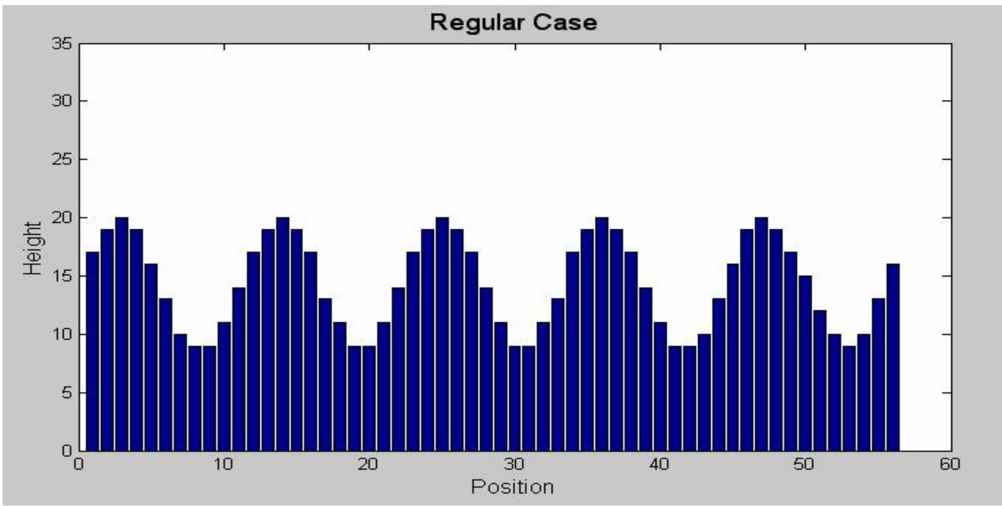


Figure. 2

The value of the x-axis indicates the index of the position of sand particles and the value of the y axis shows the number of particles in each column. Each column represents a column of sand particles, and each particle has its height, namely the value of y-axis and its position index.

Dynamics

Advection ---Observed horizontal movement of the sand model

We investigate here a parameterization of the horizontal wind stress of the form: $\mathbf{F = \alpha \quad h^p}$ **, Eq(1)**, where h = h(x), the height of sand particles at location x and **h** > 0; 0 < **α** < 1; 0 < **p** < 2. Hence, all particles in a given column will be affected by the wind.

Figure 3 and 4 show the mean height of bars as a function of the advection force strength α .

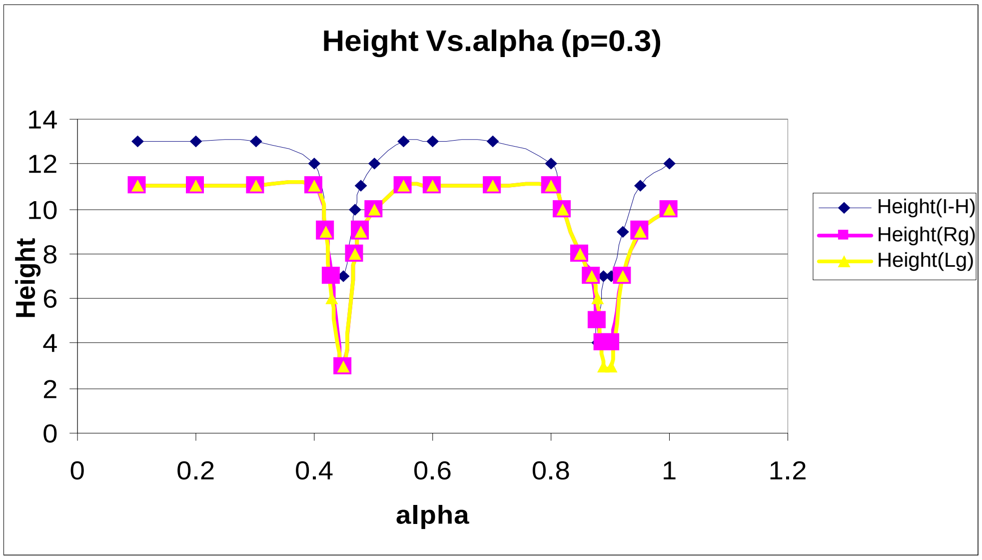


Figure 3. **Different Height vs. Alpha in combined case at p=0.3.**

Note: I-H: amplitude perturbation Rg: regular case—red line Lg: wavelength perturbation

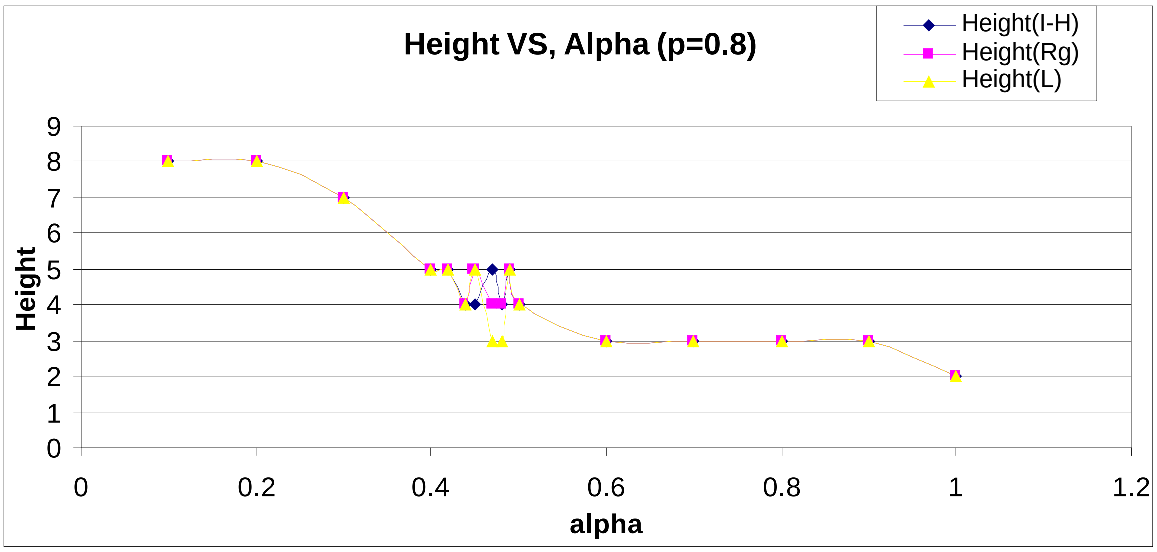


Figure 4. **Different Height vs. Alpha in combined case at p=0.8.**

Note: I-H: amplitude perturbation Rg: regular case L: wavelength perturbation

When the system comes to its stable state, there is no obvious relation between variable **α** and different height. The activations in Figure 3, and Figure 4 indicate rapid transition between bars for p=0.3, p=0.8 respectively. We set up numerical experiments with more particles as shown in Figure 5: In the configuration, we changed the distance between bars and its ratio to the height of each bar of Figure 2.

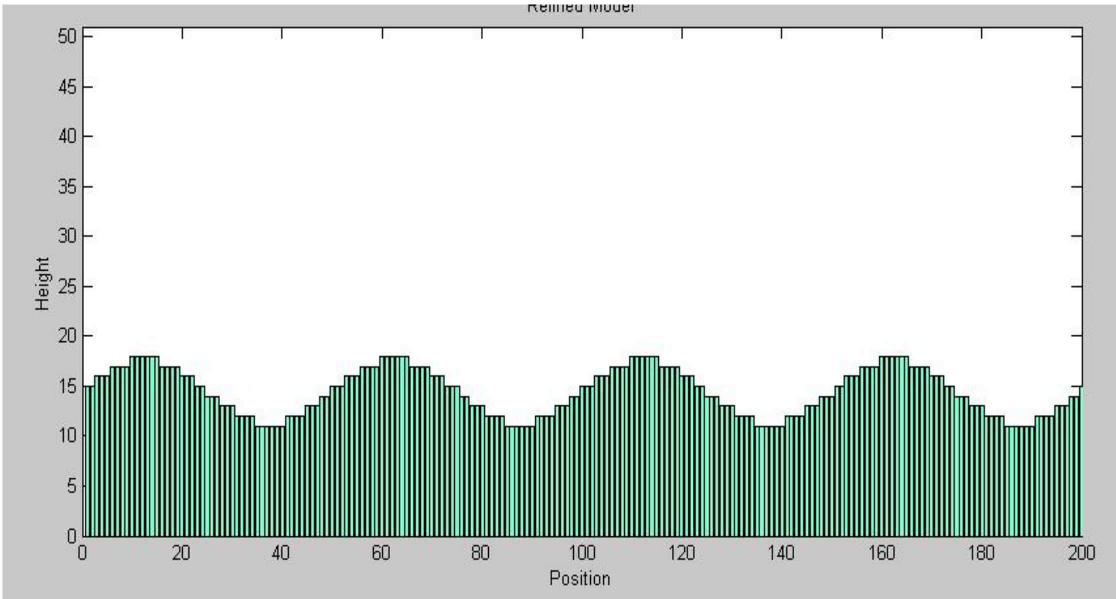
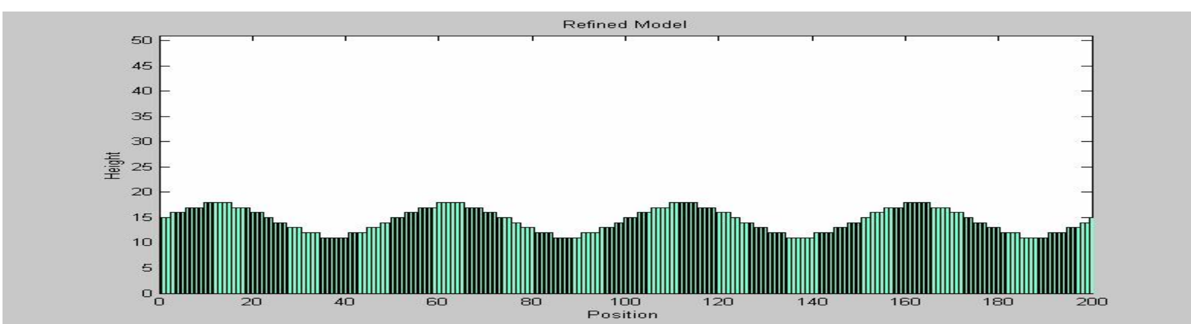


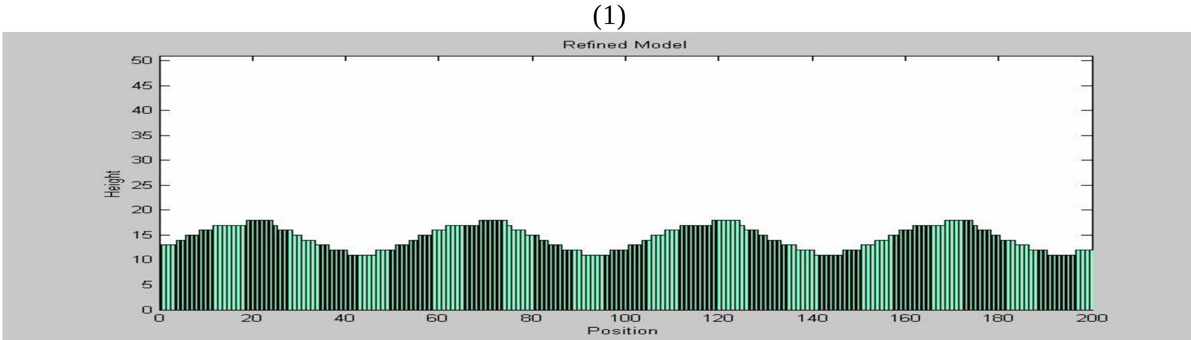
Figure .5

The value of the x-axis indicates the index of the position of sand particles and the value of the y axis shows the number of particles in each column. Each column represents a column of sand particles, and each particle has its height, namely the value of y-axis and its position index.

Then we did the same analysis as we did to the smaller experiments. Figure 6 and Figure 7 are the results we got. We applied a force and run the simulation with p = 0.3, α = 0.8.



(1)



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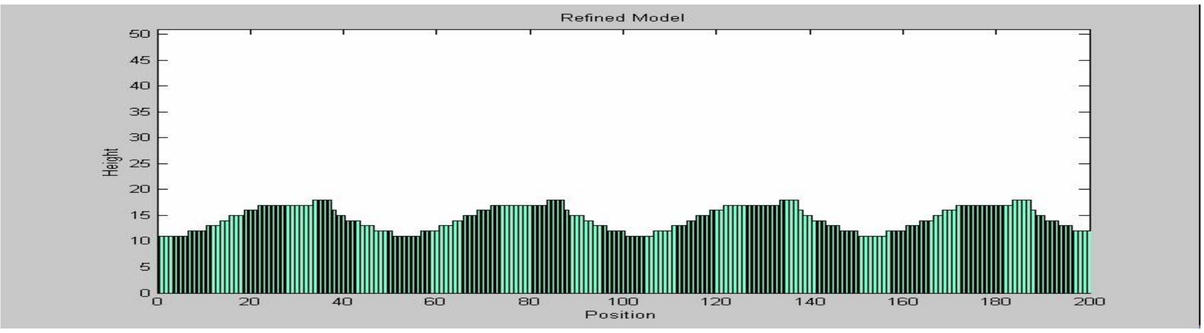

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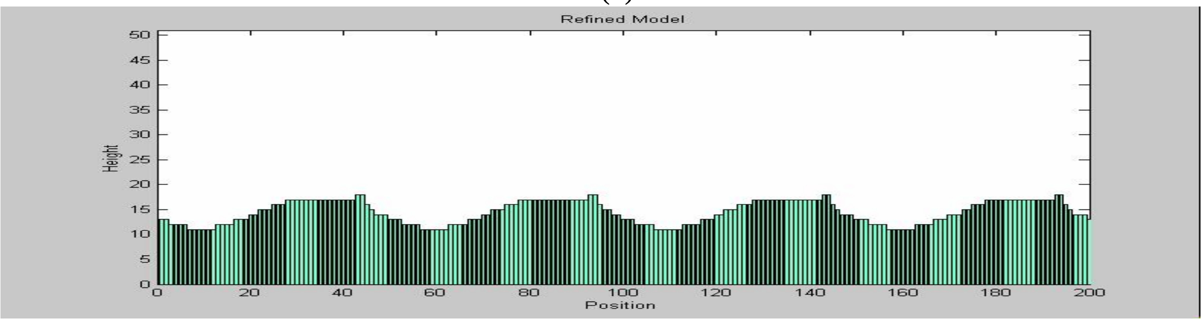

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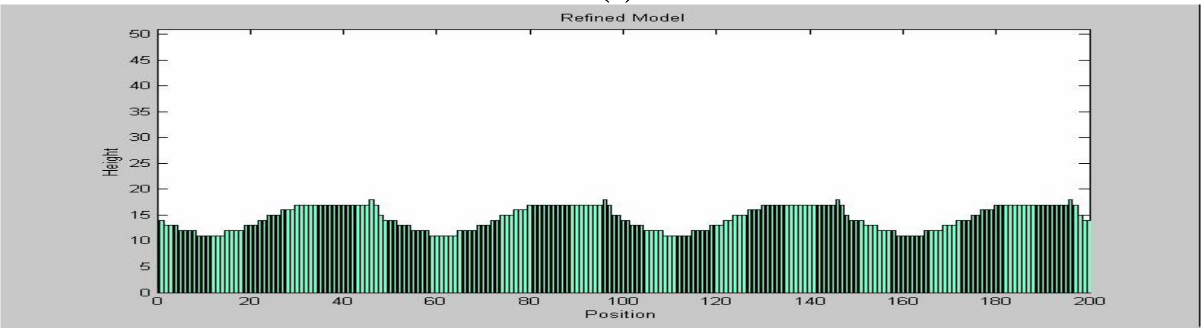

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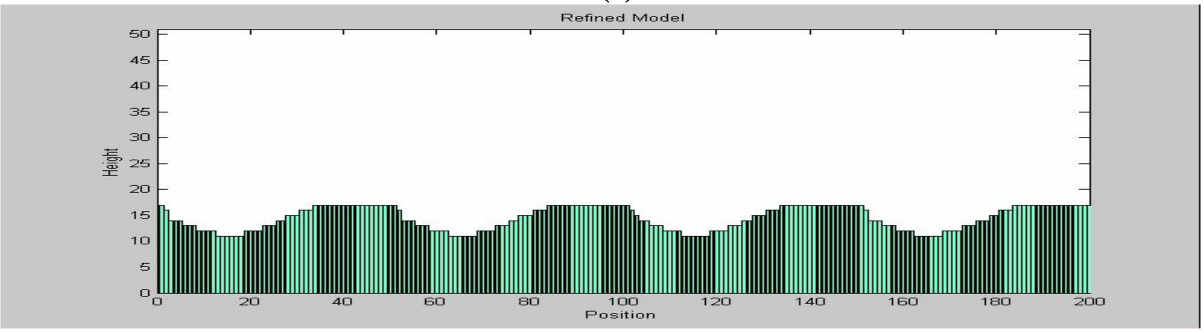

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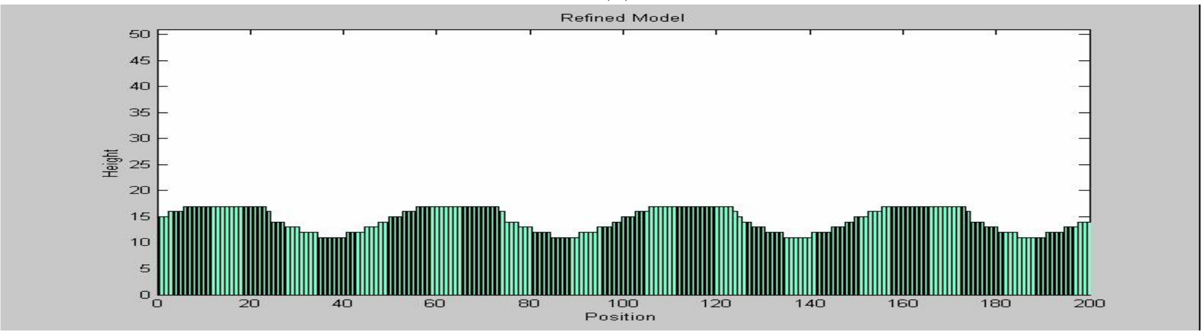


(7) Figure 6. (1) - (7)

By changing the value of alpha between 0 to1, we got the diagram of "Height vs. alpha" at p = 0.3, as shown in Figure.7

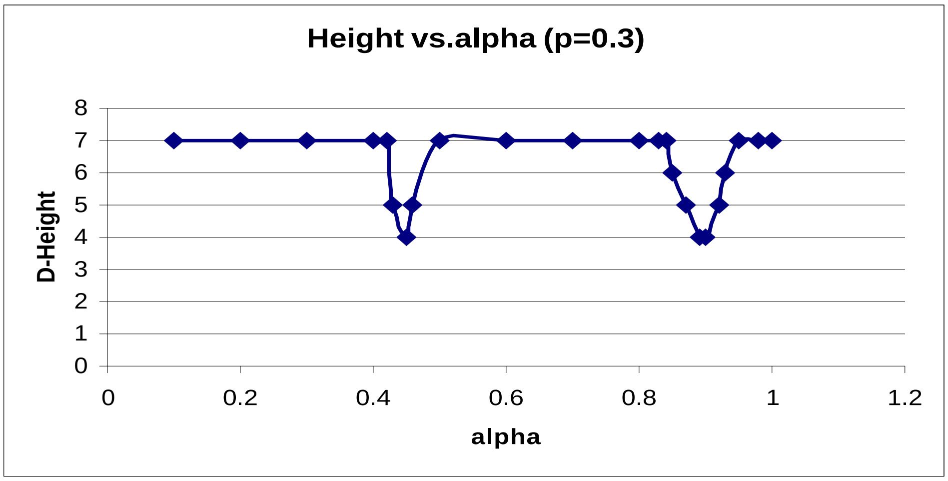
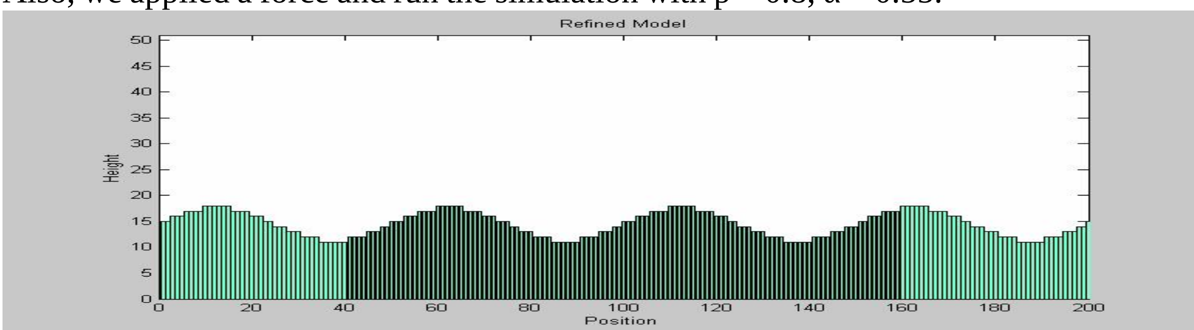
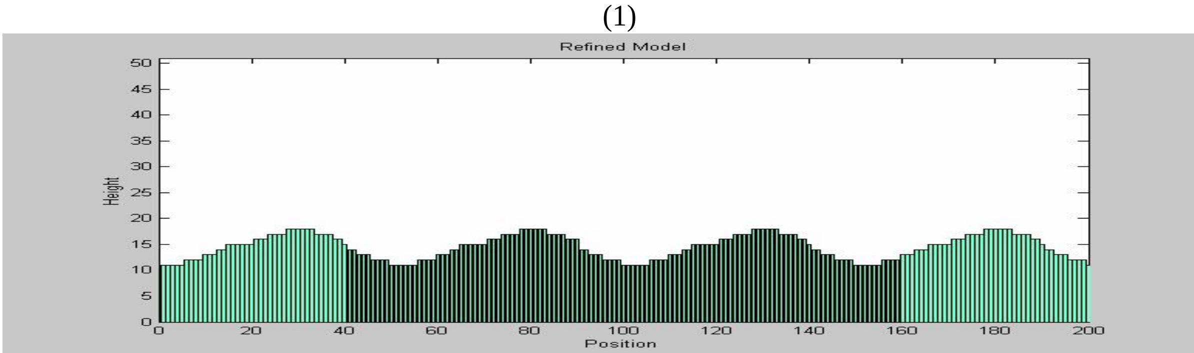


Figure.7

Also, we applied a force and run the simulation with p = 0.8, α = 0.55.



(1)



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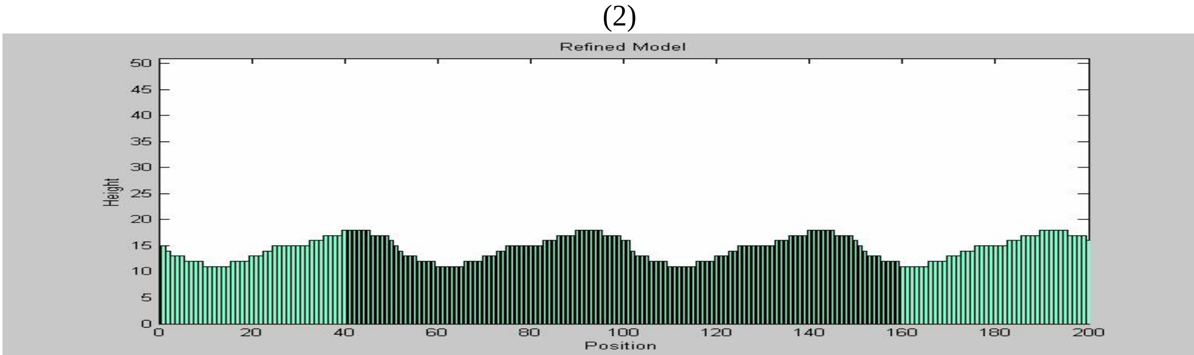

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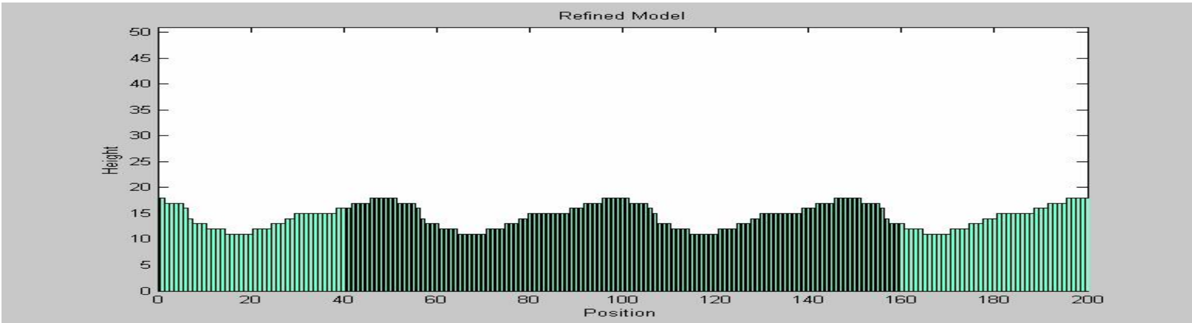
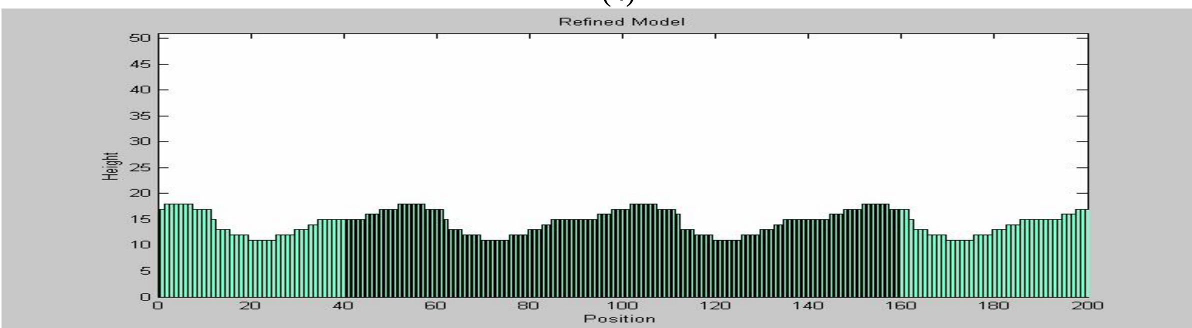

Refined Model
Height
Position

(4)


Refined Model
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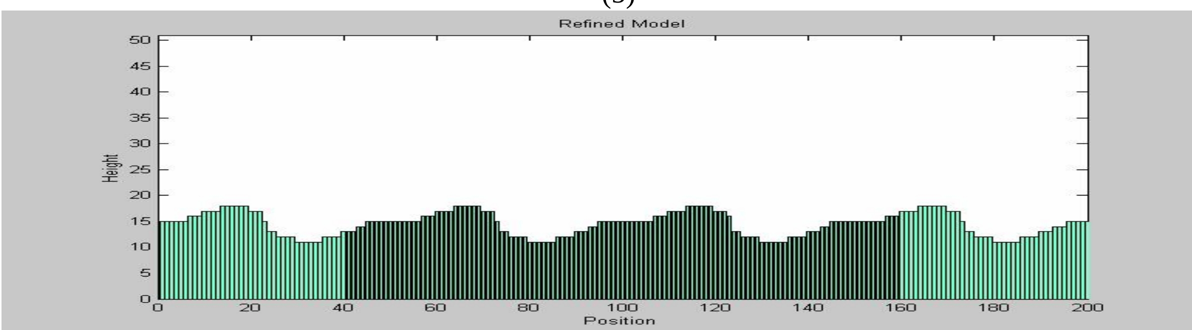
Refined Model
Height
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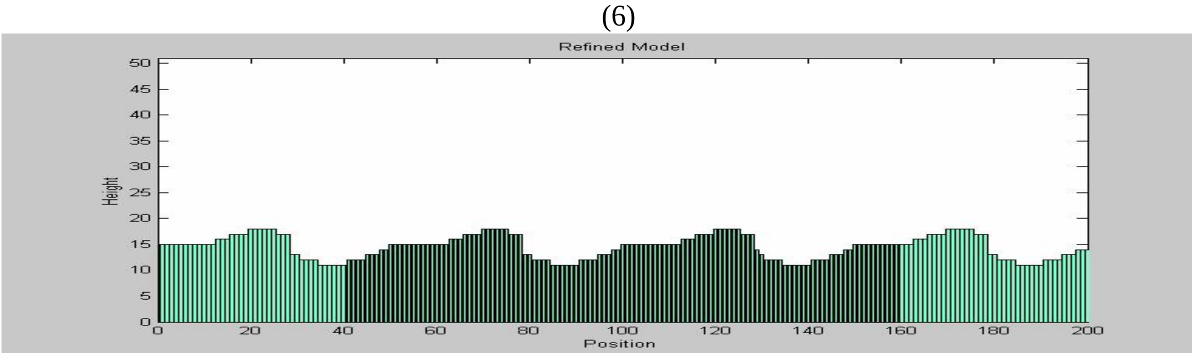
Refined Model
Height
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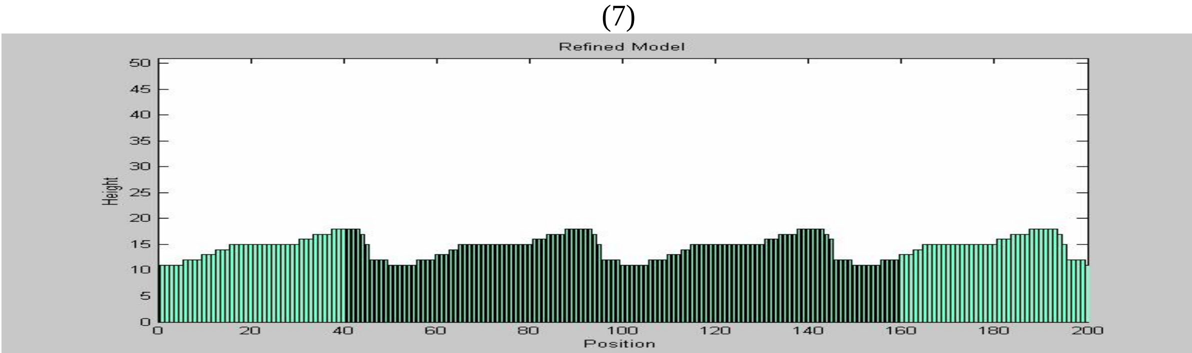
Refined Model
Height
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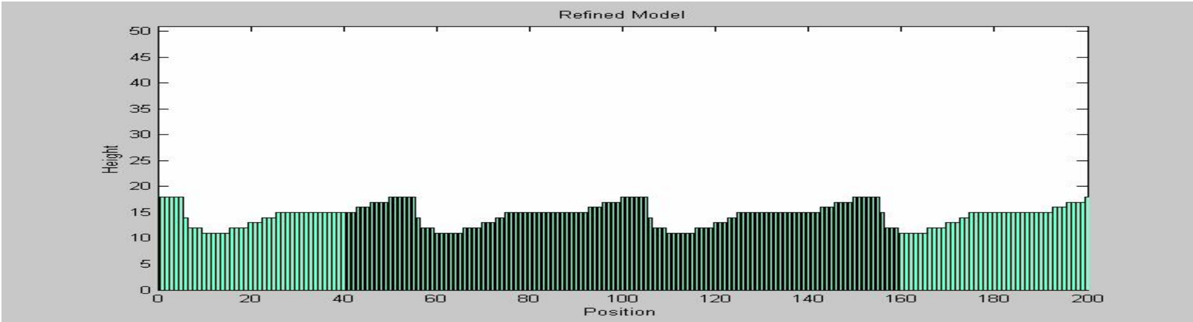
(8)


Refined Model
Height
Position
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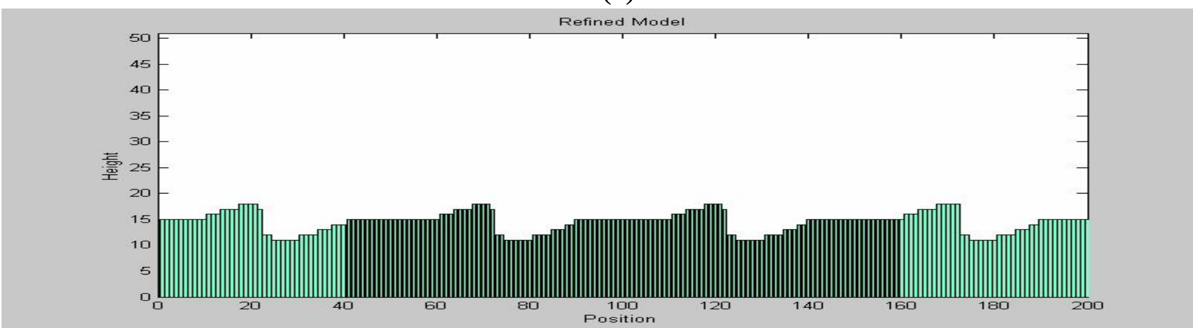
0 5 10 15 20 25 30 35 40 45 50

(9)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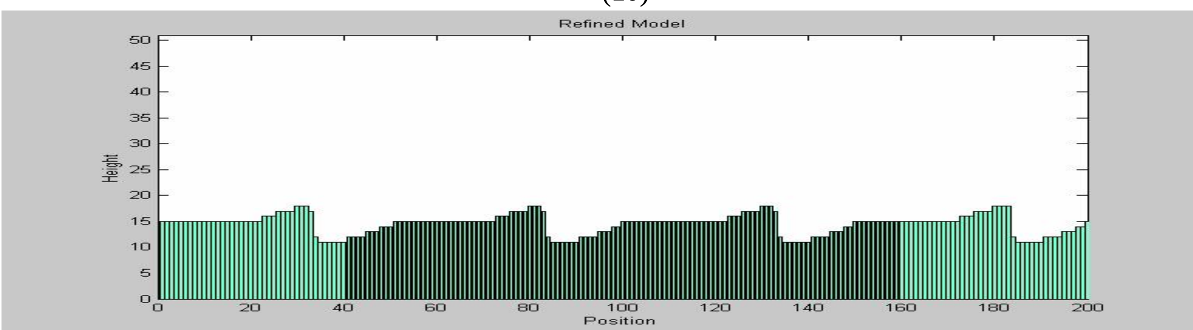
(10)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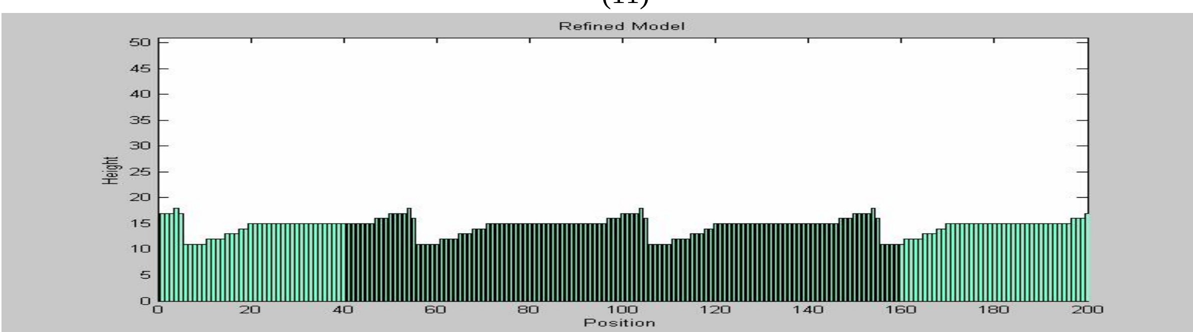
(11)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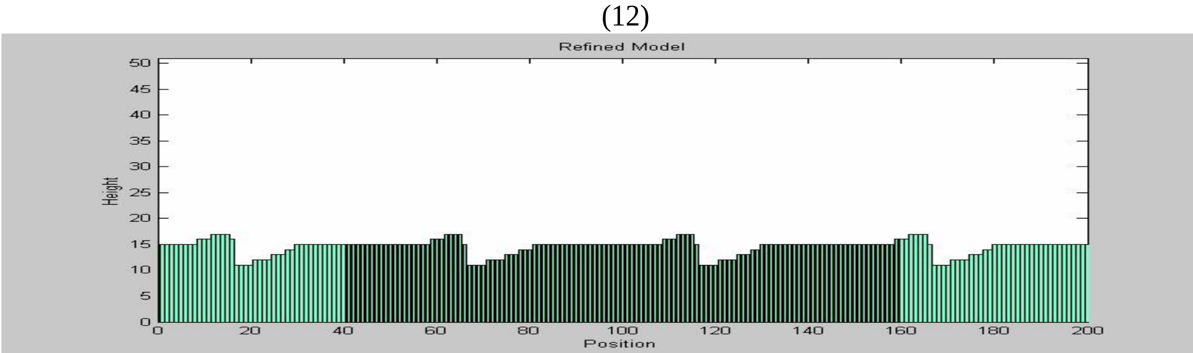
(12)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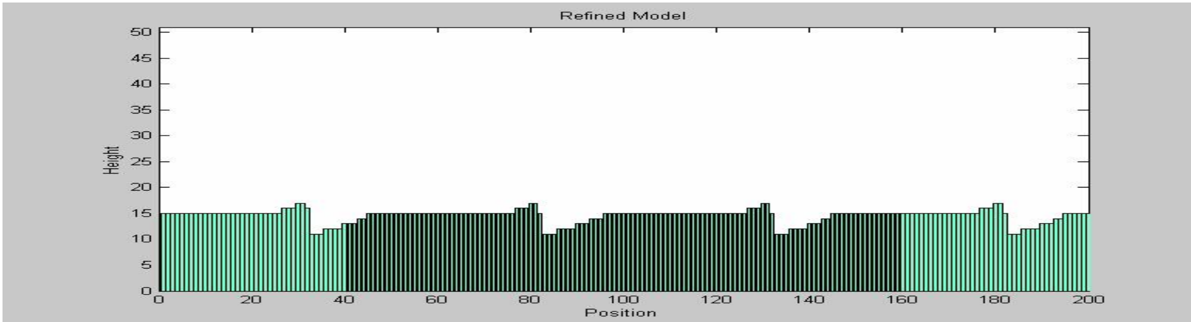
(13)



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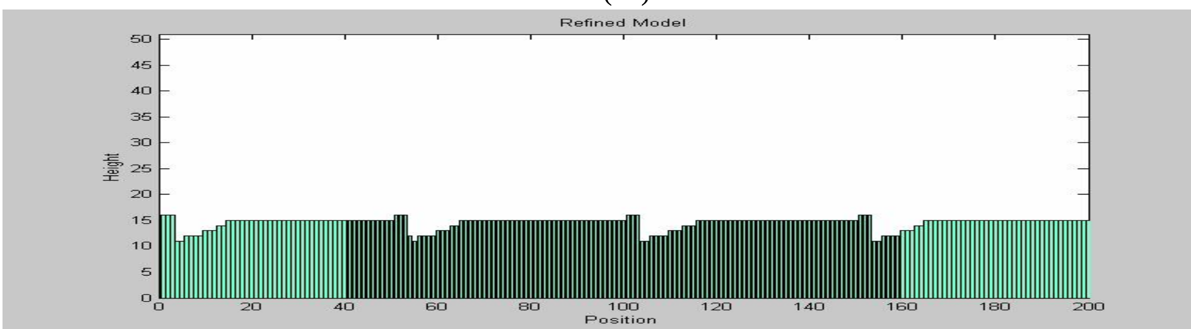

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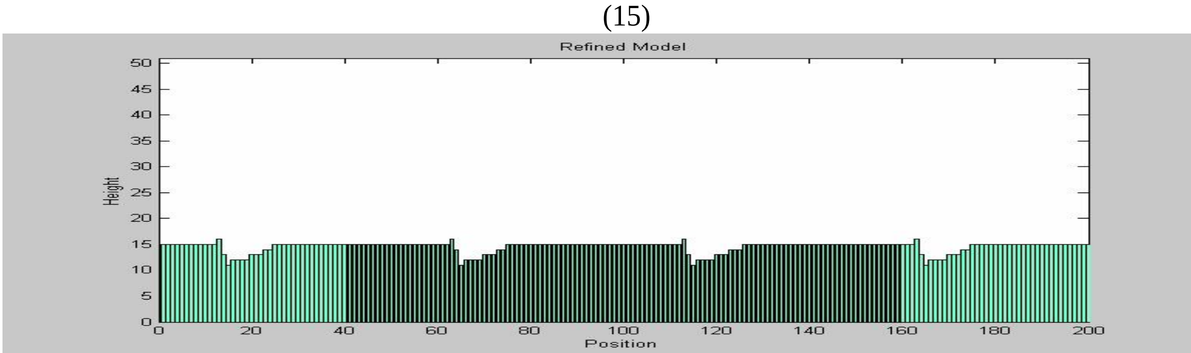

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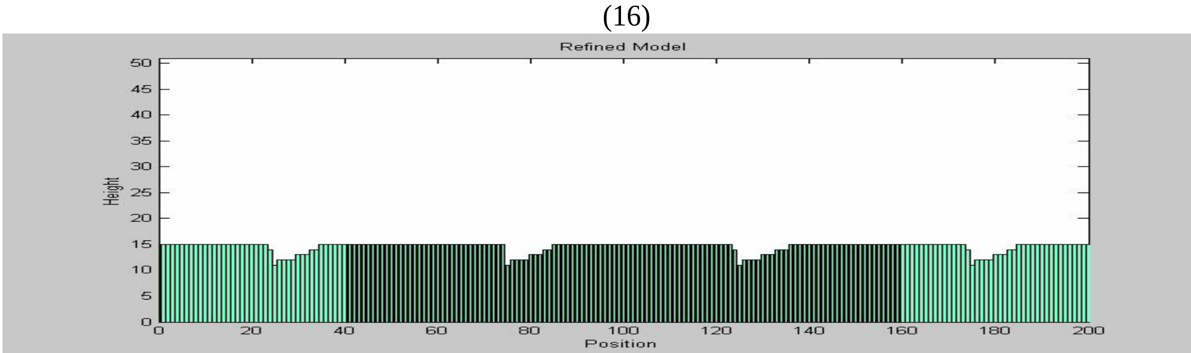

(17)

Figure 8.

By changing the value of alpha between 0 to1, we got the diagram of “Height vs. alpha” at p = 0.8, as shown in Figure.9

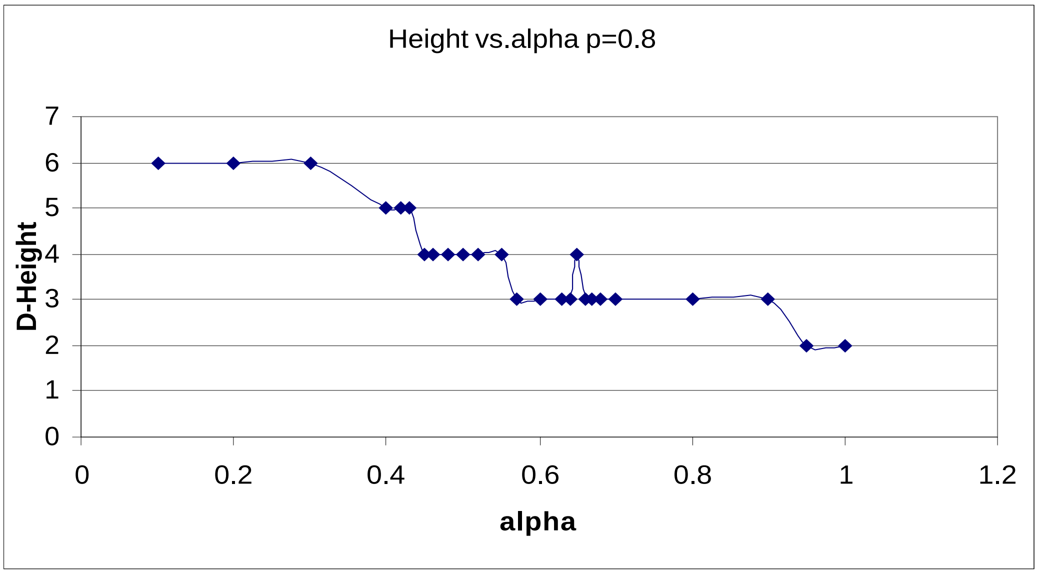

Figure 9.

Compare Figure.7 with 3, and Figure.9 with 4, results gaining from the new model have the same properties as the result of the old model. They both have transitions after simulations with the value of power factor p equals to 0.3 or 0.8.

The highly result simulation have different ratio of height vs. distance of each sand bar, comparing to the old one. But this level of variation does not change properties of sand bars system.

II. Proposed work

The Goal of present work:

We want to extend this model to 2D patterns. The model changes as well: randomly picked sand in a specified direction and deposited with a probability that depends on the local presence or absence of sand.

This work is inspired by the following sand forms: figure A, where winds or fluid flows are strong and tend to blow from one direction, and observe what affect the dune to transport and change it shape to a linear dune, such as figure B.

Figure. A [6]

Figure. B

What we want to do:

I. Simulation:

We use cellular automata and the 1-D results to foresee what a probabilistic 2D cellular automata model (shown as Figure 10) should be. Hence, we use the scheme that developed by B.T.Werner ([4], [5]).

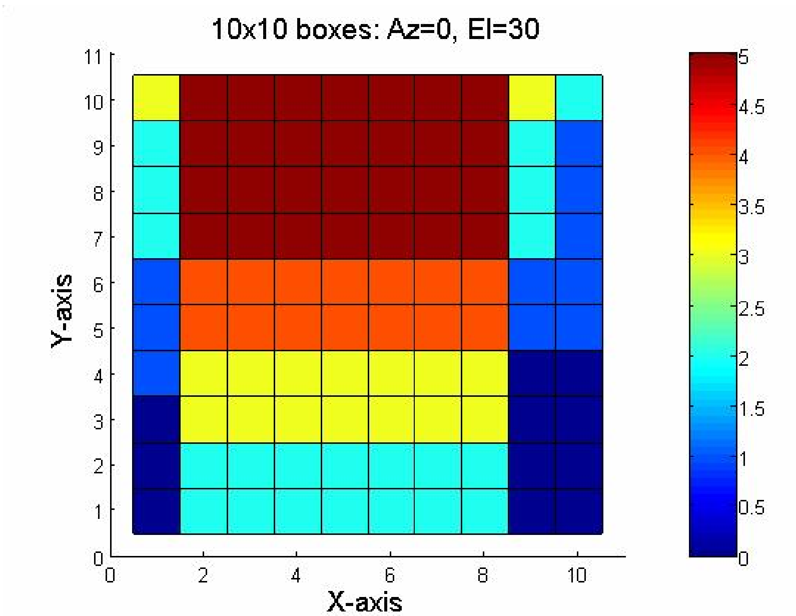

Figure 10. Each box in a lattice is presented by its 2D position: [x, y]. The different colors of boxes indicate the number of particles in each box. Az, El are view angles.

A Brief Description of Werner's 2D Model

In the computer simulation, dunes are built from slabs of sand, the position of which are constrained to lie on a square lattice (Fig .10) The surface elevation is proportional to the number of sand slabs at a lattice site. The edges of the lattice are connected by periodic boundary conditions; whereby a slab of sand transported over one boundary of the lattice is brought in at the same position on the opposite boundary. A maximum difference in surface elevation between adjacent lattice sites, an angle of response set to 30^0, as shown in Figure.11, is enforced. If the deposition of a slab of sand at a particular site would violate the angle-of-repose criterion, the slab is moved down the steepest gradient until compliance is achieves. Similarly, if erosion of a sand slab overstepping the slope, starting at that lattice site, neighboring sand slabs are moved downslope on one lattice site, successively going up the steepest gradient until no angle exceeds the angle of repose. [4]

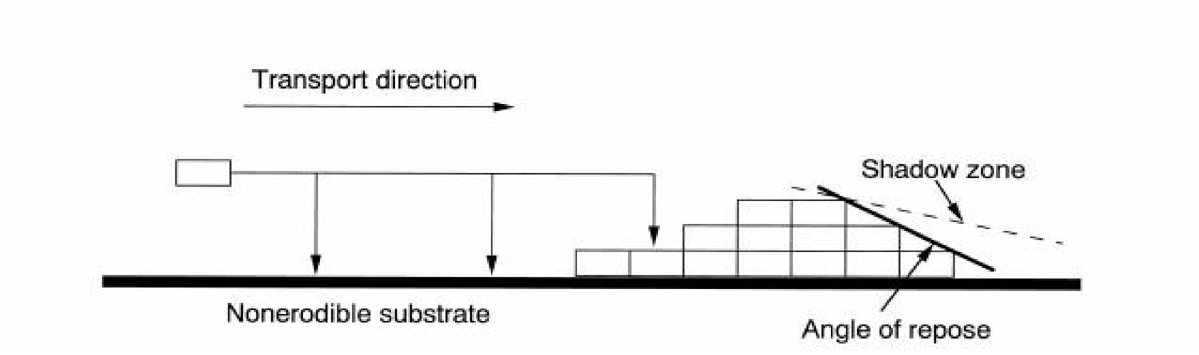

Figure.11 Werner's model of sand transport [4]

Algorithm of transportation of Werner's 2D model [4]

A sand slab is chosen for transport randomly from all sand slabs on the surface. (see Figure.11) The slab is moved a specified number of lattice sites, l , in the transport direction and is deposited at this site with a probability of deposition at a site with no sand slabs, p_{ns}, is less than the probability of deposition at a site with at least one sand slab, p_s. If the slab is not deposited, then it repeatedly is move l sites in the transport direction until deposition. This procedure is repeated to construct the time evolution of the surface.

II. Application

We use Werner's model of sand transport and apply a force which will affect the transport along both the x-axis and y-axis, and then to observe the sand dune patterns after transportations, also to explore the time of forming new sand dunes due to different forces that we apply.

References:

[1] The figure a. and b. are from Prof. Juan Restrepo**’s** laboratory experiment (work with Raymond. Goldstein & Pesci **(**University of Arizona.2002).

[2] B.H.Voorhees, Computational Analysis of One-Dimensional Cellular Automata. Vol 15, World Scientific, 1996.

[3] D.G.Green, http://life.csu.edu.su/complex/tutorials.html,1993.

[4] B.T.Werner, Eolian dunes: Computer simulations and attractor interpretation Geology, December 1995; v23; no.12; p.1107-1110.

[5] B.T.Werner, Complexity in Natural Landform Patterns Science, 2 April 1999, vol.284, p.102-104.

[6] Image from Lake City, Cañon City revisited, Great Sand Dunes, Taos. http://home.att.net/~sueno/sep02.html