Torque with Respect to Lift and Drag on a Particle in Oscillatory Flow

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1. Introduction:

The goal of this project is to determine the role played by torque on the lift and drag of a particle in a oscillatory boundary layer. This is a very practical application to fluid dynamics because these types of flow occurs everyday from chemical engineering plants to the depths of the oceans with sediment deposits. This study began by observing the lift and drag on a particle fixed to the surface of a boundary layer. In our study we raise the particle to see how it interacts with the boundary layer from above. And in the future a study on the particle that is able to move freely in the flow will be accomplished.

Since an actual experiment on a particle in oscillatory flow is next to impossible we needs to be computed numerically. These calculations are done by generating dynamics of the particle and of the moving fluid. It will be run on a beowulf cluster and the size of the calculation is: 10^6 velocity nodes and about half of that pressure nodes. We are using degree 5 spectral elements. In the code we will use a three-dimensional Navier Stokes solver along with straightforward boundary conditions. Navier Stokes is a model for an ideal incompressible isotropic Newtonian fluid, and these calculations will produce values for such things as the drag, buoyancy, and lift of particles in a highly controlled environment. Our approach will be to make measurements of the lift, drag, and buoyancy of particles by considering progressively more complex physical configurations and physics. There are three main steps that will help with in this process: we will modify an existing code to implement torque given boundary conditions; we will then test this code on known analytical results; and then we will run computational experiments to show us the role torque plays in lift and drag.

Fischer, Leaf and Restrepo (2002) completed their study on a fixed particle along a plate in oscillatory turbulent flow. Figure 4 below best describes their results. It shows that lift has favored periods where enhanced lifts are possible. This diagram shows that there is a sucking force at very short periods, or when the fluid is oscillating really fast. It also shows that at longer periods the lift is a decaying function. Although the lift has a much more interesting graph, the drag is still the dominate force, comparing the vertical axis's of the graph. This graph shows that there is high drag force when the frequency is small. They also determined that the drag force drops $1/\sqrt{\mathrm{Re}/\tau}$, where τ is the period. This study was to determine the lift and drag coefficients with respect to pressure and viscosity. For further information please refer to their study

2. Equations of Fluid Motion:

We do not have any analytical expressions for flow when a particle is present, therefore we must first describe the flow without a particle. Figure 3 shows the velocity profile on the sphere that is fixed on the plate. One can also observe that there is vortex shedding at different cycles.

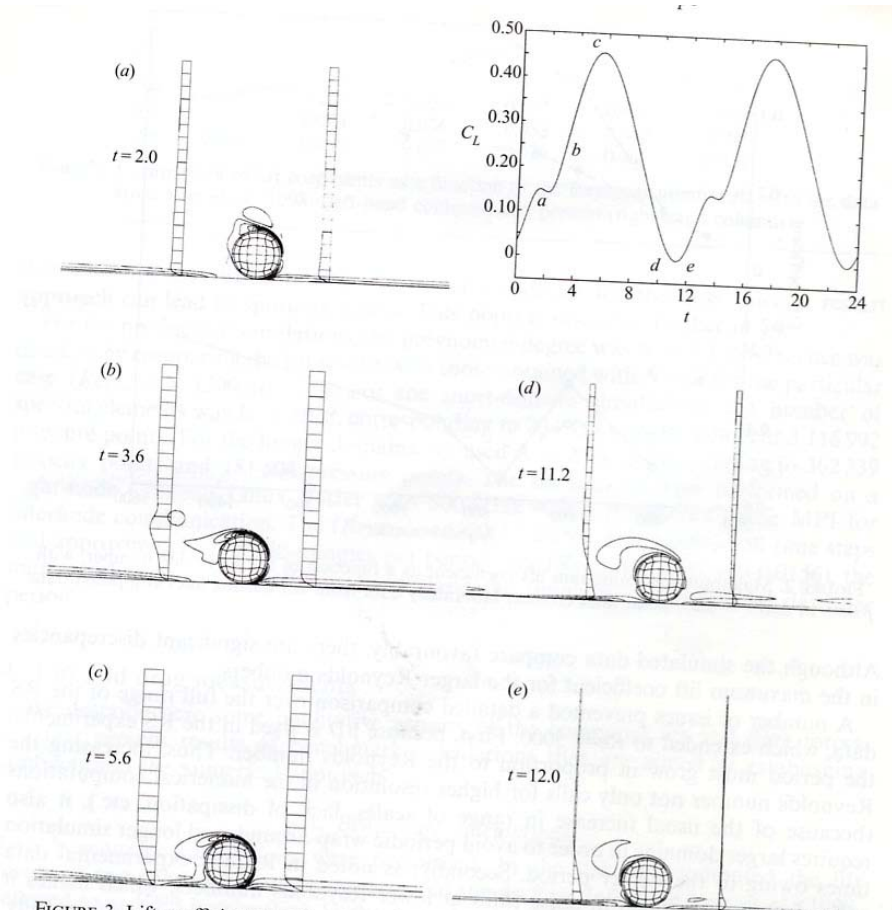

FIGURE 3. Lift coefficient as a function of time, upper right-hand corner, and corresponding vorticity distribution and velocity profiles at $x = \pm 1.5L$. $\tau = 24$, for $Re = 300$ and $\epsilon = 0.0156$.

2.1 Boundary layer flows in the absence of a particle:

First we describe the velocity of the fluid as a function of velocity in the x and y direction

$$\vec{u} = (u(x, y, t), v(x, y, t))$$

In this case we can consider that there is no velocity in the y direction. We then can use the x-direction momentum balance to determine that u is only a function of y.

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right),$$

This is a periodic process, where the velocity does not change with time during the period so the first term can be disregarded. We also determined earlier that there is no velocity in the y direction so the third term can be disregarded. In addition we are assuming that there are no pressure gradients so pressure does not change in the y-direction.

$$\vec{u} \cdot \nabla u = (u,0) \cdot (\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}) = 0$$

Then

$$0 = -\frac{\partial p}{\partial x} + \mu u_{yy}$$

The y momentum balance equation gives the following:

$$\mu \frac{\partial^2 u}{\partial y^2} = p'(x)$$

The boundary conditions for this problem are as given

$$u(y = 0) = v_0 \sin(\omega t)$$
$$u(y \to \infty) = 0$$
$$u(y, t = 0) = 0$$

The variables for non-dimensional flow are

$$y = \alpha y'$$
$$u = \beta u'$$
$$t = \delta t'$$

Where you equation now is

$$u'(y = 0) = \sin(t')$$

Now, back to our problem where the flow is not steady state and we are assuming that there are no pressure gradients. The dynamic viscosity is defined as $\upsilon = \frac{\mu}{\rho}$

The scaling factors are

$$y' = \frac{y}{\sqrt{\upsilon / \omega}} \quad , \ \alpha = \sqrt{\upsilon / \omega}$$
$$\beta = v_0$$
$$\delta = \omega^{-1}$$

This makes our partial differential equations non-dimensional.

$$\frac{\partial u'}{\partial t'} - \frac{\partial^2 u'}{\partial y'^2} = 0$$

You use your first boundary condition but change it into exponential form, because it makes for easier derivatives. We also are choosing to only use the imaginary part of the answer.

$$u'(y' = 0) = im(e^{iT})$$

We then make a guess that u is a function of g and f by separation of variables.

$$u'(y', t') = f(y) \cdot g(t)$$
$$g(t) = e^{iT}$$
$$u'(y', t') = f(y) \cdot e^{iT}$$

This then tells us that f(0)=1

$$if(y')e^{iT} = f''(y')e^{iT}$$
$$if(y') = f''(y')$$
$$f = Ae^{ry'}$$

Then we solve for the roots at take the imaginary part as our answer.

$$iAe^{ry} = r^2 Ae^{ry}u$$
$$r^2 = i = e^{i\pi/2}$$
$$r = \pm e^{i\pi/4}$$
$$r = \pm\left(\frac{1+i}{\sqrt{2}}\right)$$
$$f = A_1 e^{ry_+} + A_2 e^{ry_-}$$
$$f(y \to \infty) = 0 = f(y \to \infty)e^{iT}$$
$$therefore$$
$$A_1 = 0$$
$$f(y) = Ae^{y(-1/\sqrt{2} - i/\sqrt{2})} \cdot e^{iT}$$
$$u'(y', t') = Ae^{-y/\sqrt{2}} \cdot e^{i(T - y/\sqrt{2})}$$
$$u'(y', t') = Ae^{-y/\sqrt{2}} \sin(t - \frac{y}{\sqrt{2}})$$
$$A = 1$$

$$u'(y',t') = e^{-y/\sqrt{2}} \sin(t - \frac{y}{\sqrt{2}})$$

This is our final equation which is only the imaginary answer to the equation above.

4. Past results for lift and drag

These calculations were done on a particle that rests on an infinite plate and is subjected to a wave-induced oscillatory boundary layer flow. The velocity profile of the flow, in the absence of a particle, is described by

$$\hat{u} = U\left[\sin(\frac{2\pi t}{T}) - e^{-\beta z} \sin(\frac{2\pi t}{T} + \beta z)\right]$$

Where T is the period of oscillation and $\beta = \sqrt{\pi / \nu T}$ which is the inverse Stokes-layer thickness.

Over each time cycle one needs to record the lift and drag. These values are then used to calculate the maximum and minimum lift and drag coefficients, which are defined as

$$C_L = \frac{F_y}{\frac{1}{2} A \rho_0 U^2}$$

$$C_D = \frac{F_x}{\frac{1}{2} A \rho_0 U^2}$$

Where F_x and F_y signify the lift and drag force. F_x and F_y are calculated by finding the surface integral of the stresses over the sphere, where the stresses are pressure and viscosity. C_L and C_D are scaled in order to be non-dimensional.

4.1 Numerical approximation of the flow

Numerical solutions to this flow are based on high order spectral element code methods. We also use a third order time stepping scheme. These calculations are done at moderate Reynolds numbers and the coefficients of lift and drag are due to viscous and pressure forces. The results that Fischer, Leaf and Restrepo found are

Re	C_{LT}	C_{LV}	C_{LP}
50	0.105	0.057	0.048
100	0.074	0.033	0.041
150	0.08	0.029	0.051

C_{LT} is the total lift coefficient. C_{LV} and C_{LP} are contributions of viscous and pressure on the lift coefficient respectively.

Figure 4 shows the results of lift and drag coefficients that the past projects have determined. The graph shows the results of three different Reynolds numbers, which only vary slightly but have the same overall shape. The main results are described in more detail above, in the introduction. The period is also represented by a non-dimensional parameter, τ, where τ=TU/D. T is the period, and D is diameter of the particle.

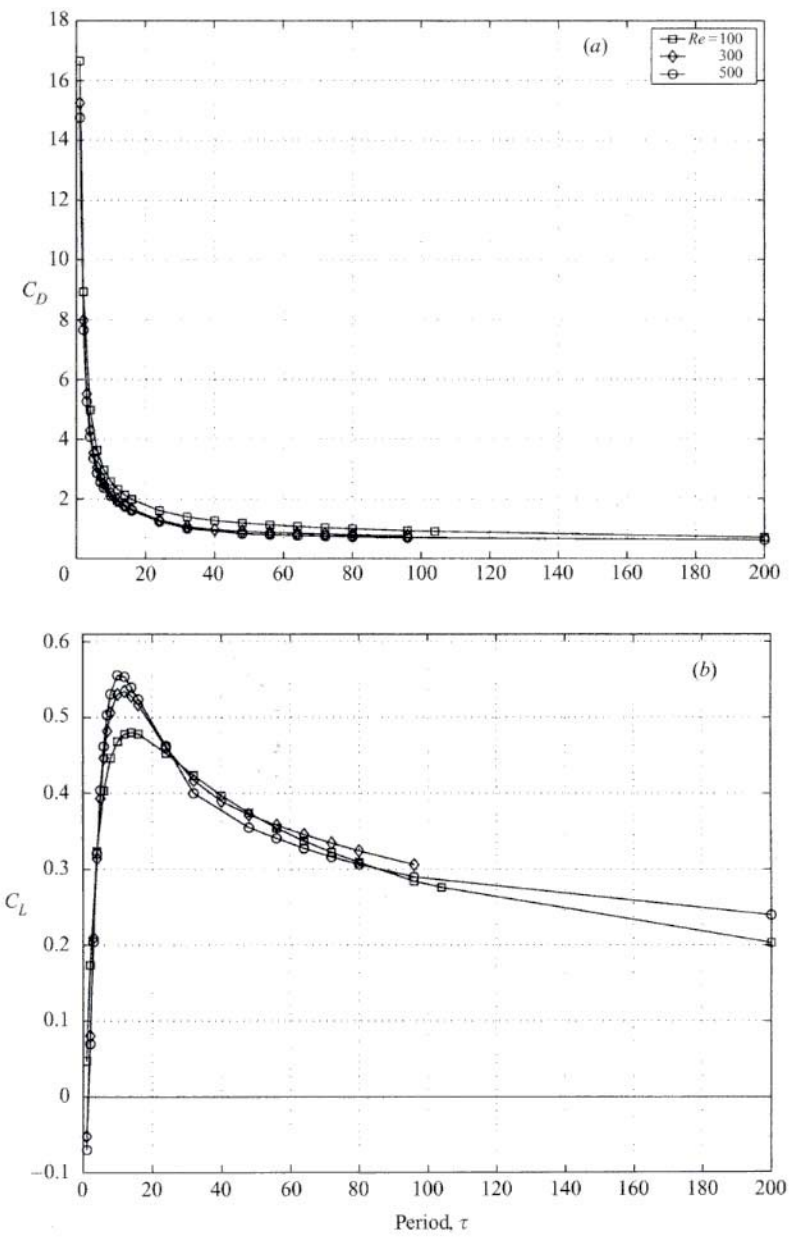

FIGURE 4. (*a*) Maximum drag coefficient, (*b*) maximum lift coefficient, as a function of period, for $Re = 100, 300, 500$. Not shown: maximum drag coefficient at $Re = 100$ for $\tau = 400$ is 0.5097, and maximum lift coefficient at $Re = 100$ for $\tau = 400$ is 0.1405.

Figure 5b shows the velocity profile in the absence of a particle. This diagram looks to similar to the equation derived from Stokes second problem earlier.

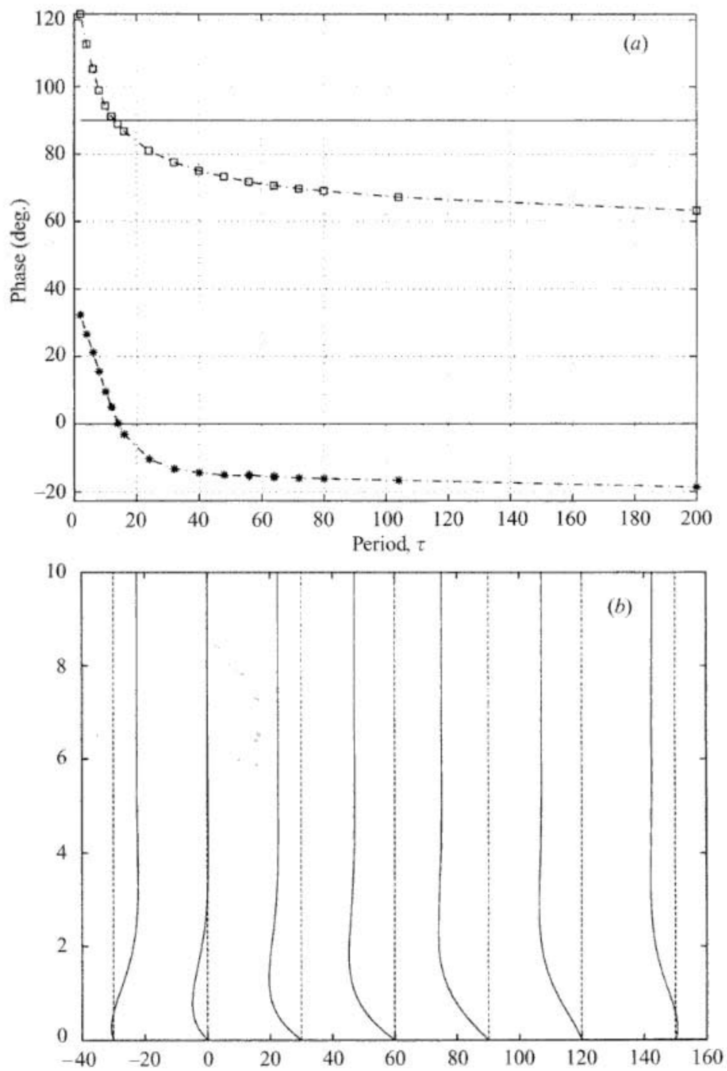

FIGURE 5. (*a*) Phase (degrees) of the peak *, minimum and □, maximum lift with respect to the forcing as a function of τ, for $Re = 100$; (*b*) velocity profile as a function of phase.

Progress:

My most recent task is to learn how to work with the mesh and the code. We are using the program preix to simulate the particle. One of the guys on our team showed us the basics of preix by building a mesh that looks like a Pacman. Although this is not the mesh for our project this image was able to let us "play" with preix to see its capabilities. Our real mesh was generated by someone at Argonne National Lab for a similar project, in which will changes theirs to fit our parameters. Our next step will be to find the appropriate boundary conditions for the wall and sphere.

I am also learning the physics behind these lift, drag and torque calculations. My main objective right now is to become skilled at running the code, building and changing the mesh, and to become comfortable using the cluster and parallel computing.

At the moment we are working on another calculation that has already been completed to verify that our code and method is correct. In this case we are looking at the interaction between two suspended spheres (Kim, Elghobashi, Sirignano). If our code is correct, our calculated value should only be off from theirs by a constant coefficient.

Future:

In the near future of this project we will hope to finish our calculation of the two spheres suspended and finalize the code. Once the code is validated, we will determine how lift and drag depend on the gap, frequency and the Reynolds number when torque forces are included. Some work will be done this summer, but the majority of the work will be done during the 2004-2005 academic year. We hope to have this project finish by summer 2005.

References:

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