

Daniel Norwood

Research Proposal for Fall 2004

Advisor: Dr. Joceline Lega

Project: Modeling of a Simple Swimmer at a Low Reynolds Number

At a low Reynolds number things don't work as one would expect, in this condition the viscosity of the medium around you has more effect on your motion than your current inertia. This condition is usually encountered in the realm of bacteria, whose diminutive mass is not great enough to gain enough momentum for conventional motion. In 1977 Purcell[1] showed that a simple one jointed swimmer would not be able to propel itself, but that what was needed was a swimmer that could undergo nonreciprocal motion.

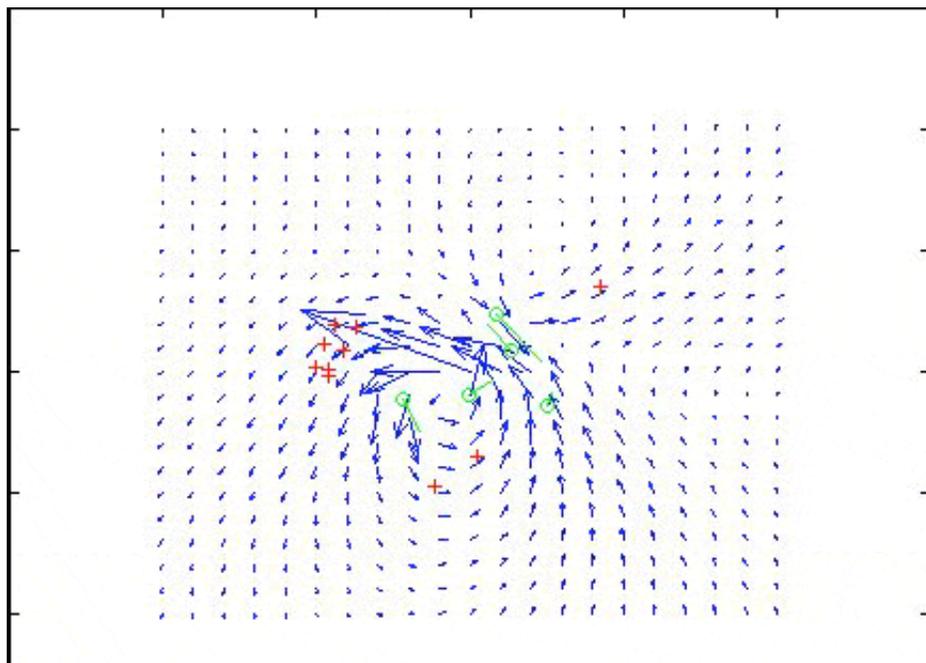
Najafi and Golenstanian[2] describe a method by which a simple swimmer comprising of three spheres connected by rigid rods can propel itself through a medium using nonreciprocal motions, as are required at a low Reynolds number. It is this simple swimmer we will be modeling.

Recently, Cortez has shown the applicability of regularized Stokeslets to simulate the motion of a stokes fluid with a given boundary[3]. In his paper he is able to numerically model the flow around various boundaries by the use of regularized Stokeslets.

Cortez has also shown how Stokeslets can be used to model flagellar motion of bacteria [4]. In this paper Cortez's team is able to numerically model and visualize the wrapping that occurs during flagellar spinning, and the resulting flow of the fluid around the body. We hope to use the same general principles, but apply them to the simple swimmer described in [2].

Over the course of this year I hope, with the aid of Dr. Lega, to work on producing a viable numerical simulation of this simple swimmer and model how it is able to interact with its environment. In doing this I will learn Fluid Dynamics and how to use the Navier-Stokes equation at a low Reynolds number as well as how to best numerically model motions in Stokes flows.

In the past few months, I have been able to get a working simulation of particles moving through a fluid, and to animate this model. So far, the model takes random points, assigns them random forces, and then models their motion.



In order to achieve this, we had to first get a model (using MatLab) of a time slice of the system. To do this, we used a regularized stokeslet as described by Cortez. We were able to simulate the motion of the point forces by incrementing the position of their stokeslet representation based on the forces acting on it. At each step we had to recalculate the forces acting on all the stokeslets, which is very processor intensive for large sets of points. At this point, our model is restricted to one so called 'blob' function:

$$\phi_\epsilon(\mathbf{x}) = \frac{3\epsilon^3}{2\pi(|\mathbf{x}|^2 + \epsilon^2)^{5/2}}.$$

In general a blob is a radially symmetric and smooth function where  $\text{Int}(\phi(x)dx) = 1$  [3]. These are used to 'spread' the forces over an area so as to get rid of any singularities that may occur.

I have also begun setting up a system of equations to generate a boundary in the simulation. This is important because any swimmer we wish to model is just a collection of moving boundaries. To do this, we must solve the matrix equation  $U=MF$  where  $U$  is a  $2N$  vector of velocities,  $F$  is a  $2N$  vector of forces, and  $M$  is a  $2N \times 2N$  matrix consisting of the  $(x,y)$  coordinates of our desired boundary. Each element of  $U$  is given by the equation:

$$(11) \quad \mathbf{u}(\mathbf{x}) = \sum_{k=1}^N \frac{-\mathbf{f}_k}{4\pi\mu} \left[ \ln \left( \sqrt{r_k^2 + \epsilon^2} + \epsilon \right) - \frac{\epsilon \left( \sqrt{r_k^2 + \epsilon^2} + 2\epsilon \right)}{\left( \sqrt{r_k^2 + \epsilon^2} + \epsilon \right) \sqrt{r_k^2 + \epsilon^2}} \right] + \frac{1}{4\pi\mu} [\mathbf{f}_k \cdot (\mathbf{x} - \mathbf{x}_k)] (\mathbf{x} - \mathbf{x}_k) \left[ \frac{\sqrt{r_k^2 + \epsilon^2} + 2\epsilon}{\left( \sqrt{r_k^2 + \epsilon^2} + \epsilon \right)^2 \sqrt{r_k^2 + \epsilon^2}} \right],$$

where  $r_k = |\mathbf{x} - \mathbf{x}_k|$ .

[3]

So to create the correct matrix equation we need to split (11) into a matrix equation by taking the force component and putting it into the  $F$  vector. This leaves us with the matrix  $M$ , a matrix that is usually not invertible. This is because we are trying to set the sums of the scalar parts of the equation to equal 0. By doing this, we make the Matrix singular. I will have to use a method such as gmres to solve for this matrix.

Over the course of this semester I plan to generalize the current system we have. I want to be able to plug in any 'blob' function and have the system react appropriately. I also plan on generating a system whereby the forces on the stokeslets do not remain constant, but may vary by either a time sensitive variable, or by external factors in the system. Another current problem in the system is that there is no easy way to align our point forces along a given boundary, as would be necessary to model our simple swimmer. I plan on implementing a method of easily placing any number of points along a given boundary.

[1] Purcell, E.M. "Life at Low Reynolds Number."

[2] Golestanian, R.; Najafi, Ali, "Simple Swimmer at low Reynolds Number: Three Linked Spheres."

[3] Cortez, R. "The method of Regularized Stokeslets."

[4] Cortez, R.; et al. "A Study of Bacterial Flagellar Bundling."