

Proposal for a Mahler Measure Project

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1 Introduction

The *Mahler measure* is a property of polynomials with integer coefficients. Specifically, it is the product of the absolute values of those roots of the polynomial which, viewed in the complex plane, lie strictly outside the unit circle [1].

The Mahler measure actually enjoys many equivalent definitions, and it is scrutinized in such diverse areas as dynamical systems, K-theory, number theory, topology and analysis [2]. The measure has intimate connections with the cyclotomic polynomials. For example, a polynomial has Mahler measure 1 iff it is a product of cyclotomics and the monomial x [3]. An open conjecture of Lehmer claims that there is a constant c such that if a polynomial's Mahler measure is smaller than c , then that measure is 1.

2 Goals of the Project

Our goal is to explore ways of systematically generating polynomials with small Mahler measure. This would lend insight into Lehmer's conjecture. A stronger form [4] of Lehmer's conjecture gives a specific c and a specific polynomial with Mahler measure c , and so it is remotely possible that we could unearth a polynomial with Mahler measure nontrivially smaller than c ; this would be a major breakthrough.

3 Plan of Attack

One way to generate polynomials with interesting Mahler measures is to use trigonometric identities. D. Madden suggests we attempt to generalize this attack and instead use identities involving Jacobian elliptic curves. Jacobian elliptic curves are in fact a generalization of trigonometry and enjoy a vast wealth of interesting properties [5],[6]. This avenue of attack on Mahler measures is quite unexplored, and for this reason it's not unreasonable to hope for some original insight.

4 References

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