Research Proposal for Undergraduate Research Assistantship

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This project focuses on a formula published recently by Wendelin Werner [1] which gives a measure on self-avoiding loops in the complex plane. By Self-avoiding loops one means closed loops that do not intersect themselves. Werner proves in his paper the existence of this measure, which is unique (up to multiplication by constants) and invariant under conformal transformations.

A measure assigns a nonnegative number to a subset. Depending on the application, the measure of a subset can be interpreted as its physical size, the amount of elements that lie in the subset or the probability that a random process will yield a result within the subset. For example the measure of an interval (a, b) in the real line could be the length of the interval $b - a$. Also, the measure of the union of two disjoint measurable sets must be the sum of their measures. The formula presented by Werner states that for $D' \subset D \subset \mathbb{C}$ and for a $z \in D'$ the measure is given by $\mu_{D'} = c \log \Phi'(z)$ where $\Phi : D' \to D$ is a conformal map that fixes z [1].

The focus of this project will be restricted to a particular subset of self-avoiding loops in a subdomain $D' \subset D$ where D is the the unit disk centered at $z = 0$ and D' is D minus the segment from r to 1 where $0 < r < 1$. The purpose of the project will be to find a conformal map Φ and its derivative Φ' for this particular class of loops, which is required for the formula provided by Werner to be used on them. The map must satisfy two conditions: map the origin to itself, and the derivative of the map must be positive. The formula provided by Werner will then be compared with computer simulations of the self-avoiding loops generated by the advisor.

Initially, the student will focus on learning about conformal maps and their properties, and learn how to apply the formula. Progress has been made in the effort to find the desired conformal map Φ. Conformal maps are transformations from some domain in the complex plane to another domain. One characteristic of these maps is that they preserve angles locally (e.g. the angles formed by the tangent lines of two curves at their point of intersection.) Conformal maps are useful in the study of flows and fields and in solving boundary-value problems [2]. Self-avoiding loops arise in applications of statistichal mechanics [1].

References

[1] Werner, Wendelin. The Conformally Invariant Measure on Self-Avoiding Loops. arXiv:math/0511605v3

[2] Fisher, S.D. Complex Variables. Second Edition. Dover. 1999