Quantum Information and Teleportation

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Quantum mechanics, despite its lack of intuitiveness, has been very successful in describing and predicting physical systems. The field of quantum computation and information seeks to use quantum mechanics to do the same for information processing. In quantum mechanics, electron spin is described by a two state system, spin up and spin down. Similarly in quantum information there are two basis vectors called qubits which are analogous to classical bits of information.

$$|0\rangle = (1,0)$$
$$|1\rangle = (0,1)$$

Where $|0\rangle$ and $|1\rangle$ are column vectors. Quantum mechanics allows for a superposition of basis states, so the qubit can be described by the state

$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Where α and β are complex numbers. The qubit $|\Psi\rangle$ is then a vector in a 2D complex vector space. According to the postulates of quantum mechanics, the only measurement outcomes are 0 with a probability $|\alpha|^2$ or 1 with a probability of $|\beta|^2$. When this measurement is taken, the superposition collapses to the state $|0\rangle$ if 0 is observed or to the state $|1\rangle$ if 1 is observed. Therefore the original state of the qubit is changed with only a single bit of information obtained. The nature of quantum mechanics then presents a major challenge to information processing. A qubit has the potential to represent an infinite amount of information, but as long as it remains unobserved. How then can a qubit be transported while recovering its original state? The answer is quantum teleportation. Quantum teleportation uses the experimentally verified concept of quantum correlation or entanglement. In quantum entanglement, a measurement on one of a pair of states affects the other, even if the pair is spatially separated. A pair of qubits would have four basis states, described by their linear combination.

$$|\Psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$$

In the quantum teleportation, the qubit to be transported is interacted with a shared pair of qubits. The qubit is transported by means of quantum gates, as well as with the use of classical bits of information.

The first part of this project will show how quantum teleportation has been achieved with maximally entangled pairs of qubits, also called Bell pairs $|\beta_{xy}\rangle$. It will also be proved that if the shared pair is in a less than maximally entangled state, teleportation is only possible with a certain probability less than one, which is a function of the entanglement coefficient. This requires the techniques of linear algebra, specifically quantum measurement formalism and Schmidt decomposition.

$$|\beta_{xy}\rangle = \frac{|0,y\rangle + (-1)^x|1,\bar{y}\rangle}{\sqrt{2}}$$

Where $\bar{y}$ is the negation of y.
If time permits, the second part will investigate teleportation in quantum networks or lattices, currently a new field, using the techniques of percolation theory.

Bibliography:
Nielsen, Michael A. and Isaac L. Chuang. Quantum Computation and Quantum Information. Cambridge, U.K.; New York: Cambridge University Press, 2000.