The Algebra Initiative Colloquium

Volume 2

Papers presented at a conference on reform in algebra
December 9–12, 1993

Edited by

Carole B. Lacampagne
William Blair
Jim Kaput

U.S. Department of Education
Office of Educational Research and Improvement
National Institute on Student Achievement, Curriculum, and Assessment

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Suggestions for the Teaching of Algebra

William Yslas Vélez
University of Arizona

As I looked over the material that was sent to me, I decided to focus in on three different topics.

- Sociology of the classroom,
- Use of technology in the classroom, and
- Course content.

Sociology of the Classroom

The notion that we have to have more cultural diversity in mathematics based careers is a topic that has been given a great deal of press recently. It is an absolute necessity that all individuals in this country have access to these careers. But how is this lofty goal to be achieved? That is the real question.

I am not going to emphasize this question in this paper. I think that we have a more basic problem in this country, namely, how can we attract *all* of our own children into the study of mathematics? I have to admit that I do not have clear ideas as to how to encourage our own children to avoid the culture of drugs and alcohol that is so pervasive in this country and instead choose a more reflective lifestyle. Even as I write this sentence, I am overcome by the ridiculousness of this statement. Our children are being taken away from us by a culture of violence, and we appear almost powerless to entice them from this.

It is this point of view that flavors my comments. I am not going to focus on how to attract those very bright children into the study of algebra. Instead, I want to think of how we can attract all of our children into being mathematically literate, so that they will have the opportunity to participate in the present technological revolution. I believe that this task requires not only a rethinking of what it is that we teach but also how it is that we present that material.

As I attempt to encourage the Chicano students in the Southwest to consider mathematics—based fields, I am struck by one fact. Whatever it is that I do to entice these students into these studies has little to do with the fact that they share the same cultural heritage as I, but rather it has everything to do with a simple dictum that my mother used to tell me, *Hablando se entiende la gente,* that is, "By speaking we make ourselves understood." I do not think that the mathematics community has made a very concerted effort to inform our populace that mathematics is not only an exciting field of study, but it also leads to interesting careers that carry with them attractive monetary and intellectual rewards. In order to emphasize this point, I would like to make some personal remarks.

I grew up in that poverty that was part of the culture of being a Chicano in the Southwest. This poverty and a Mexican-American up-bringing created culture shock as I was forced to function in the American school system. There are differences in the cultures, differences which sometimes make living together difficult. With this in mind, allow me to recount the following incidents. On two separate occasions the following occurred. I had called a colleague at his home in order to pick up a book or something. It was agreed that I could come over and pick it up. When I arrived the colleague opened the door, told me that he would get the object for me, then closed the door in my face as he retreated into his home to get the object. This person had invited me over and left me standing outside. I was shocked by this behavior, and I have often told myself that no Hispanic would behave in such an uneducated fashion as to leave a guest standing on his/her doorstep. These two incidents have always stood out in mind as I have thought of the interactions of the Mexican-American and the Anglo cultures. In the Mexican culture, the term *educado* carries with it not the implication of a formal education but rather the connotation of a person who is cultured enough to treat others with dignity and respect. From my own cultural perspective, I often felt that Anglo academics were not *educados* in spite of their "education."

I mention these incidents for the following reasons. I believe that the community of mathematics professors invites students into their classrooms and then closes the door and leaves these students on the doorstep, mostly in shock and feeling demoralized. I am not saying that Anglo professors leave minority students in the cold, though that may be true. I am saying that mathematics professors leave most students there.

One of the reasons for this behavior is cultural. The mathematics profession is a culture inhabited by mostly very bright people, or so we want the rest of the world to think. In order to maintain this perception, we must behave in ways that would encourage the population to believe this. We are rational human beings whose professional goals are to think about this very difficult subject, mathematics. We teach our classes with a certain aloofness so that students won't bother us. There is no question that there is a certain facade to our profession, a facade that makes communication difficult.

But it is also true that we are abstract thinkers, and pretty good at it. In fact, those who end up teaching mathematics at universities are the survivors of the training process for the profession; they are the ones who are good at it. So how do we communicate this to our students? Communicate what? This abstract thinking is such a natural part of our being that it does not occur to us most of the time to even think of this aspect of teaching. Yet this is also a great cultural difference between ourselves, as professors, and the students who must not only struggle with the new concepts but also with this new way of thinking.

In summary, I would make the following recommendations:

- Communication is the key to education. Besides teaching our students we should take time to speak with them about mathematics and the career opportunities for mathematics-based fields.
- Our main job as teachers of mathematics is not to pass on to the students a technique, but rather a point of view.
- We should recognize that mathematicians are abstract thinkers, and good at it. However, the student population is not as adept at it as we are, and they must be trained.
- We should attempt to teach the students who show up in our classrooms. It is pointless to compare these students to the past or to wish for the students of the future.

Technology in the Classroom

In the fall semester of 1992, I had the good fortune to teach first semester calculus using the Harvard Calculus Consortium notes. The leading philosophy of the course is the Rule of Three, that all ideas should be presented numerically, geometrically, and analytically. This philosophy has made me think about how it is that I present material, and it also forces me to think how it is that I could use technology to better present the material. The profession has barely begun to scratch the surface here. I think that we are at the beginning of a real revolution.

At the same time that I began to teach this course, I also began using the IBM Personal Science Laboratory (PSL). The PSL connects to a computer in the classroom and allows the instructor to run experiments in the classroom. The data from these experiments is projected onto a screen as the experiment is running. This provides an immediate connection between the experiment and the data. The PSL comes with probes for measuring distance, temperature, and light intensity, among others. The distance probe allows one to measure the distance of a moving object. This piece of technology allows the instructor to generate data immediately, display the data geometrically, and then fit a curve to the data. This is a wonderful example of the Rule of Three.

If I am to address the issue of technology in the classroom, then I cannot address this issue in abstract terms, but rather I must discuss my own very limited experiences.

Knowing that I had the computer technology available for my first semester Calculus class, I gave the following assignment to my students. I asked the students to devise two experiments which I would be able to run using the PSL. One of the experiments was to produce data that was linear and the other was to have data that was quadratic. I fully expected that the quadratic data was to be very easy, simply drop a ball. I was quite surprised to find that many of the students did not know that a falling object should have a quadratic path. This assignment had been given more than halfway into the course, and we had had a couple of homework problems dealing with a quadratic equation modeling a falling body. Yet many of the students still had not internalized this fact. In discussions that I had with the students, I found out that some thought that the data should be exponential. But even here there was disagreement; some said that it should be e^(kt), while others thought it should have the form a*(1-e^(-kt)), where k is positive. I was shocked, yet here was reality.

I corrected the students' thinking on this topic and pointed out that falling bodies should have quadratic paths. So now students gave me all kinds of experiments, drop a ball or bounce a ball off a wall. I pointed out that I had to be able to run the experiments in the classroom, and dropping a ball would take less than three seconds, not enough time to gather decent data. Besides, I didn't want to break the probe by dropping a ball on it. Finally, we discussed rolling objects down an inclined plane. I told the students that they had to bring in things to roll down. It was wonderful. Students brought in skateboards, bowling balls, basketballs, and pairs of rolling skates tied together, but the results of the experiments were disappointing. The data did not look quadratic enough. As a ball was let go down an inclined plane, the data looked almost linear. I was panicking as I had the students looking at me as I ran these disappointing experiments in front of them, and then it occurred to me! Why not roll the ball up the plane and let it come down. The data was dramatic, a perfect parabola.

Though these experiments were in a Calculus class, I think it appropriate to discuss them here, as we address the issue of presenting algebra. After all, a polynomial, one of the basic ingredients of algebra, is being used to model a very real situation.

Though I have taught calculus and linear algebra courses that used either a graphics calculator or used some specific software, I have not had enough experience to make solid recommendations. I am convinced that the intelligent use of technology will serve to make mathematics instruction more effective. I would hope that in the future all of our mathematics courses have a laboratory associated to them.

Summarizing, I would make the following recommendations:

- The mathematics community should aggressively seek new ways of presenting the material. The use of technology in the classroom should be encouraged at all levels of instruction.
- Change does not necessarily occur spontaneously. Our faculty have to be encouraged to seek new ways of instruction.

Content of Our Algebra Courses

Algebra is the study of structure, yet this is probably news to the undergraduate student. To most of our students, algebra is equated with calculation, and this calculation is the servant of science, not a subject of interest in itself. This same attitude is reinforced in our Calculus classes.

We no longer teach structure in high school or in Calculus. Perhaps this is one of the reasons that the transition from lower division mathematics to our abstract upper division mathematics courses is so difficult. We emphasize calculations at the beginning of the training of our mathematics undergraduates, and then change the rules on these students when they make the transition to the upper division mathematics courses. These lower division students think that they are doing well in their studies, only to find out that the major that they chose has suddenly become almost unrecognizable to them as they begin studying abstract and linear algebra and advanced calculus. We have tried to soften this transition by introducing a proof course, but these courses have had little success.

Perhaps the first instance in a student's career where structure cannot be ignored is the study of differential equations. In many of these sophomore level courses, ideas from linear algebra are introduced in order to be able to explicitly obtain solutions to classes of differential equations. This infusion of linear algebra is not done because of the love of algebra, but rather because the structure of the solutions can no longer be ignored.

It is my own belief that linear algebra is the most important upper division course that a student can take. The methods and results of this subject pervade all aspects of mathematics and its applications. The subject introduces itself early in the high school curriculum, makes itself indispensable at the sophomore level, in a context completely different form its nascency, and then, phoenix-like, rises again to motivate so much graduate mathematics and applications. If we are looking for an algebra thread to weave through a student's education, from high school through graduate studies, what better than the subject of linear algebra?

One problem with linear algebra, at present, is that the subject almost dies through three semesters of Calculus. Now here is a challenge. Is it possible to bring this subject of linear algebra into our Calculus course in a meaningful way?

The other mainstay of algebra at the upper division level is, of course, modern algebra. Many universities offer a two semester course in this subject. The first semester is usually the standard course covering the topics of groups, rings, and fields. The second semester can be more of the same abstract development or it can be a course in applied algebra. I would prefer the second course for the following reason. For those students taking a second course in abstract algebra, which would cover theoretical topics such as Galois theory, I would assume that these students are going on to graduate school. If so, the students will cover this material in greater depth at a higher level, so why have two semesters of this course? I would prefer a second course to cover a variety of applications, for example, coding theory, cryptography, crystallographic groups, or finite fields. Perhaps a second course could focus on algorithmic aspects of algebra, covering Grobner bases, for example. Such an applied course has the benefit that it can draw in a wider audience and attract students from engineering and computer science.

In summary, I would make the following recommendations:

- College algebra should reflect the transition from our knowledge in high school to a higher level of sophistication.

- Linear algebra should be the thread that goes through mathematics, from high school through graduate school.
- A course on the applications of algebra should be offered instead of a second course in abstract algebra.
- In all of these subjects, the use of technology should be pursued in order to better present these abstract ideas.
- It is laudable that we have attempted to make calculus more intuitive. Proofs have been replaced by an appeal to geometric intuition or numerical calculations. Students leave calculus with a better understanding of the basic ideas of calculus. Since most of our calculus students will not go on to become mathematics majors, I think that these students leave with a better understanding of mathematics. However, some of these students will go on to take upper division mathematics courses. In these upper division courses, proof still reigns supreme. Proof is what makes our science. Proof is our tool for sharpening our intuition and for carrying out our investigations. Proof cannot be ignored. The challenge is then to find a way of bridging the gap that exists between the way we present lower division mathematics to the way we present upper division mathematics.