8.3 (d) Show that $(0, 1)$ and $(0, \infty)$ are equinumerous.

We need to find a function $f : (0, 1) \to (0, \infty)$ which is bijective.

We will try

$$f(x) = \frac{1}{x} - 1.$$

Need to check that this is a surjective 1-1 map from $(0, 1)$ to $(0, \infty)$.

8.3 (e) Show that $(0, 1)$ and $\mathbb{R}$ are equinumerous. Try

$$f(x) = \tan(\pi x - \pi/2)$$

An algebraic example would be a good idea, because then we wouldn't have to prove properties of the tangent function.

8.3(b) Show that $[0, 1]$ and $[0, 1)$ are equinumerous

Define $f : [0, 1] \longrightarrow [0, 1)$ by

$$f(x) = \begin{cases} x & \text{if } 0 \leq x < 1, \text{ and } x \neq 1/2^n,\ n \in \mathbb{N} \\ 1/2^{n+1} & \text{if } x = 1/2^n,\ n \in \mathbb{N} \end{cases}$$

8.4 Show that $(0,1)$ and (m,n) are equinumerous

We have to find a function $f:(0,1)\to(m,n)$.

$$f(x)=(n-m)x+m$$

Suppose we have two intervals (m,n) and (m',n') and each is eqninumerous with $(0,1)$ then each is equinumerous with the other because equinumerositudinousness is an equivalence relation.

Very cool fact. Last time we saw that $(0,1)$ has uncountable many real numbers. Now we know that every interval, no matter how small, has uncountably many numbers in it.

We are given $f : S \setminus T \to T \setminus S$ which is 1-1 and onto. Want to define $g : S \to T$

$$g(s) = \begin{cases} f(s) & \text{if } s \in S \setminus T \\ s & \text{if } s \in S \cap T \end{cases}$$