Solving Equations

William McCallum

Add something at beginning collecting the rules of arithmetic, and summarizing the discussion of equations, giving definition of equation and solution.

Solving as a process of mathematical reasoning

How would you respond to this as a teacher?

$$\begin{array}{rcl} x^2 - 3x - 4 & = & 0 \\ x^2 - 3x & = & 4 \\ x(x-3) & = & 4 \\ x = 2 & \text{or} & x - 3 = 2 \\ x = 2 & \text{or} & x = 5 \end{array}$$

What is a correct solution?

$$x^2 - 3x - 4 = 0, \tag{1}$$

$$(x+1)(x-4) = 0. \tag{2}$$

$$x + 1 = 0 \quad \text{or} \quad x - 4 = 0, \tag{3}$$

$$x = -1 \quad \text{or} \quad x = 4.$$

How do you justify the steps? How do you get students to see the difference between correct steps and incorrect ones?

Now let's try to justify the steps. Before we do the justification, let's decide exactly what it is we are trying to justify.

When we claim to have solved an equation we make two claims: that the numbers we have found are solutions, and that there aren't any other numbers that are solutions.

The mystery of what x is disappears when we put it into a complete sentence: "If x is a number such that $x^2 - 3x - 4 = 0$, then $x = -1$ or $x = 4$. "

Statement to be proved:

- The numbers $x = -1$ and $x = 4$ make

$$x^2 - 3x - 4 = 0.$$

- These are the only two numbers that make the equation true.

Solving the equation

The zero factor principle

We want to use the rules of arithmetic to prove that if $AB = 0$ then $A = 0$ or $B = 0$.

Suppose that $AB = 0$. Then either $A = 0$ or $A \neq 0$. If $A \neq 0$ we can multiply both sides of this equation by A^{-1} to get

$$\begin{aligned} AB &= 0 \\ A^{-1}(AB) &= A^{-1} \cdot 0 \\ (A^{-1}A)B &= 0 \\ 1 \cdot B &= 0 \\ B &= 0 \end{aligned}$$

Wrap up

Does what you have learned in this lesson affect the way you would teach quadratic equations?

You may not give all these details to your students, but unless you understand the logical reasoning behind procedure you don't have the ability to evaluate incorrect solutions, nor to recognize unconventional correct solutions.