Conceptual Understanding and Procedural Fluency: the Exponent and Logarithm

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The exponent rules

Exponent rules

$$x^n x^m = x^{n+m} \qquad (x^n)^m = x^{nm}$$

$$x^0 = 1 \qquad x^{-n} = \frac{1}{x^n}$$

$$x^{1/n} = \sqrt[n]{x} \qquad x^{n/m} = \sqrt[m]{x^n}$$

How do we help students remember all this rules? Often rote memorization goes awry. For example, if students are looking at an expression like ab^5, they can get mixed up and think that a is also raised to the 5th power.

Definition of the logarithm

1	2
2	
3	8
4	
5	32
	64
7	128

What is the pattern of the left column? Fill in the blanks of the left column. What is the pattern of the right column? Fill in the blanks there. If 9 is in the left column, what is in the right column? If 64^3 is in the right column, what is in the left column?

Now consider the following table. What goes in the columns on the left?

1	2
	3
2	4
	5
	6
	7
3	8
	9
	10
	11
	12
	13

1.0000	2
1.5850	3
2.0000	4
2.3219	5
2.5850	6
2.8073	7
3	8
3.1699	9
3.3219	10
3.4594	11
3.5850	12
3.7004	13

How long we can expect the left hand column to start with a 3? How long will the 4's last?

Definition
The logarithm (to base 2) of n is the exponent that you raise 2 to to get n.
In symbols:

$$\log_2 n = p \quad \text{if and only if} \quad 2^p = n.$$

Does this definition explain the repetition of the 5850 and the 3219 in the digits to the right of the decimal point in the table?

Logarithm rules

Tell me some facts you know about logarithms

1. $\log_2(AB) = \log_2 A + \log_2 B$
2. $\log_2 1 = 0$
3. $\log_2(A^p) = p \log_2 A$
4. $\log_2(2^x) = x$
5. $2^{\log_2 x} = x$
6. $\log_2(1/x) = -\log_2 x$

Proving the logarithm rules using the definition

How do you get students to remember these?

For example, $\log_2 1 = 0$ because 0 is the exponent that you raise 2 to to get 1. Why? Because $2^0 = 1$. Can you explain the others? Which is more useful, the chart or the definition?