

The role of the impulse to solve equations

- 1) Whole numbers  $x+a=b$  | Addition  
 $(x+40=20)$
- 2) Integers  $ax=b$  | Multiplication
- 3) Rational numbers  $x^n=b, b>0$
- 4) Roots  $f(x)=b, f$  a continuous function
- 5) Real numbers  $a^x=c$
- 6) Complex numbers  $x=\log_a c$

$$\begin{cases} 2x=3 \\ x=3 \end{cases} \text{ same solution}$$

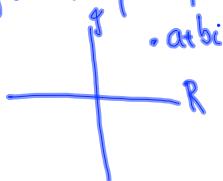
$a^b \neq b^a$   
So solving  $x^b=c$  solution  $x$  is not  
is not the same as solving  $a^x=c$  solution is a log  
 $x=c^{1/b}$   
 $x=\log_a c$

Complex numbers

$$x^2 = -1$$

Adding in the solution to  $x^2 = -1$  to the number system creates solutions to every polynomial equation

$$x^4 = -1$$



$$x^4 = -1 \quad x^2 = \pm i \quad x + iy = x' + iy' \\ x = \sqrt{\pm i} \quad \Leftrightarrow x = x' \quad (x, x', y, y' \in \mathbb{R}) \\ y = y'$$

$$(a+bi)^2 = i$$

$$a, b \in \mathbb{R} \quad a^2 + 2ab i + b^2 i^2 = i$$

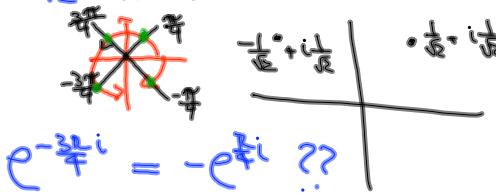
$$\text{imp } \frac{\pi}{4} \quad (a^2 - b^2) + 2ab i = i = 0 + i$$

$$a^2 - b^2 = 0 \quad \rightarrow a = \pm \frac{1}{\sqrt{2}}, b = \pm \frac{1}{\sqrt{2}} \\ 2ab = 1 \quad \left( \frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}} \right)$$

$$-(x+iy) = -x-iy$$

$$e^{i\frac{\pi}{4}}, e^{-i\frac{\pi}{4}}, e^{i\frac{3\pi}{4}}, e^{-i\frac{3\pi}{4}}$$

$$\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}$$



$a + bi$

$$r \cos \theta + i(r \sin \theta) ??$$

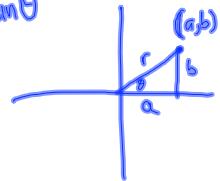
How do you convert between  
these forms?

$$e^{i\theta} = e^{\theta} e^{i\theta}$$

$$r e^{i\theta} = (\cos \theta + i \sin \theta)$$

$$a = r \cos \theta$$

$$b = r \sin \theta$$



$$(\cos \theta + i \sin \theta)(\cos \phi + i \sin \phi)$$

$$???$$

$$\equiv \cos(\theta + \phi) + i \sin(\theta + \phi)$$

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$$e^{i\frac{\pi}{4}} \quad \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$$