In terms of the equation $y = 2^x$ explain why the "vertical tail" of graph appears to shift to the right by 10 every time the y-range is multiplied by 1000.

$2^{10} \cong 1000$

So x increases by 10 y increases by a factor of 1000

Shift of $y=f(x)$ to the right by a is $y=f(x-a)$

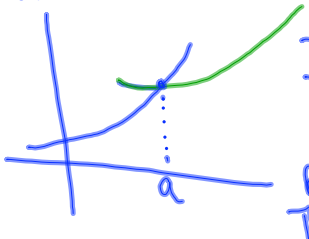
Look at $\frac{2^x}{1000}$ and 2^{x-10}

$2^x 2^{-10} = \frac{2^x}{2^{10}} \cong \frac{2^x}{1000}$

Why 2^x beats x^{10}

Calculus

Suppose two functions start at the same value. $\lim_{x\to\infty} \frac{x^{10}}{2^x} = 0$



If $f(a) > g(a)$

$f'(x) > g'(x)$

for all $x > a$

Then $f(x) > g(x)$ $\forall x > a$

Use this idea to prove 2^x beats x (ie $2^x > x$ for all x past a certain point)

To prove $2^x > x^{10}$ for all x past a certain point, keep taking derivative of x^{10}

$10x^9$, $10 \cdot 9 \cdot x^8$, $10 \cdot 9 \cdot 8 \cdot x^7$, ..., $10!$ (10! · x)

Idea for argument

2^x x^{10}

$(\ln 2)^8 2^x$ $\frac{10!}{2} x^2$

$(\ln 2)^9 2^x$ $10! x$

$(\ln 2)^{10} 2^x$ $10!$