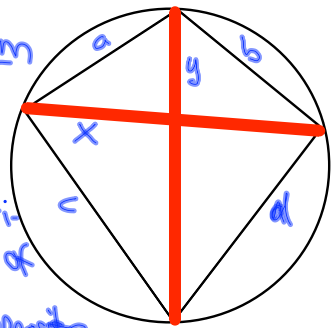
$$x^2 = \frac{(ad+bc)(ac+bd)}{(ab+cd)}$$

$$y^2 = \frac{(ad+bc)(ab+cd)}{(ac+bd)}$$

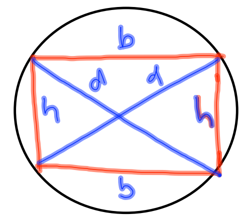
$$x^2y^2 = (ad+bc)^2 \qquad xy = ad+bc \text{ !!!!}$$

Ptolemy's theorem

The product of the diagonals of a cyclic quadrilateral is the sum of the product of opposite sides.



Ptolemy's theorem for rectangles



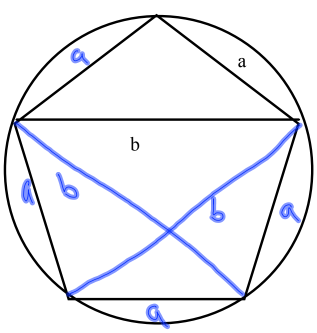
$$d\cdot d = h\cdot h + b\cdot b$$

$$d^2 = h^2 + b^2$$

is Pythagoras's theorem

$$b^2 - ab - a^2 = 0 \qquad b = \frac{a \pm \sqrt{a^2+4a^2}}{2} = \left(\frac{1+\sqrt{5}}{2}\right)a$$

Find the ratio between a and b



$$b^2 = a^2 + ab$$

$$\left(\frac{a}{b}\right)^2 + \left(\frac{a}{b}\right) - 1 = 0$$

$$\frac{a}{b} = \frac{-1+\sqrt{5}}{2} = \frac{1}{\frac{1+\sqrt{5}}{2}}$$

$$\frac{a}{b} = \frac{-1+\sqrt{5}}{2} = \frac{1}{2\cos\left(\frac{\pi}{5}\right)} = \frac{1}{2\sin\left(\frac{3\pi}{10}\right)}$$

$$= \frac{1}{\sqrt{2\left(1-\cos\left(\frac{3\pi}{5}\right)\right)}}$$

$$\cos\left(\frac{\pi}{5}\right) = \frac{1/2}{\left(\frac{-1+\sqrt{5}}{2}\right)} = \frac{1+\sqrt{5}}{4}$$

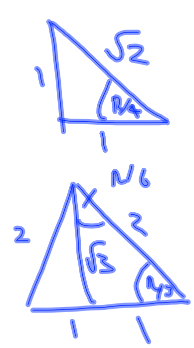
$$\sin\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\tan\left(\frac{\pi}{4}\right) = 1$$

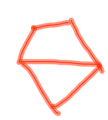
$$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$\vdots$$

$$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$



$$\cos\left(\frac{\pi}{5}\right) = \frac{1+\sqrt{5}}{4}$$



If two triangles have the same area and the same perimeter, are they congruent?

- Counterexamples (if you think false)
- Some neat way of parameterizing triangles (if true)