

1) Bill drives 400 miles at 50 miles per hour. For how long does he drive?

2) Kiera bought some CD's and DVD's

She bought 14 disks and paid \$184.

If CD's cost \$12 and DVD's \$20 how many of each did she buy?

3) Tony has a jar with nickels, dimes and quarters. The nickels and dimes total \$40. The dimes and quarters total \$55. The nickels and quarters total \$35. How many of each does she have?

$$1) \quad 400 = 50t$$

$$t = 50^{-1} \cdot 400 = 8$$

$$2) \quad \begin{aligned} D + C &= 14 \\ 20D + 12C &= 184 \end{aligned}$$

$$\begin{aligned} -20D - 12C &= -168 \\ 8D &= 16 \end{aligned}$$

$$\begin{aligned} -20D - 12C &= -168 \\ D &= 2 \end{aligned}$$

$$\begin{aligned} -12C &= -144 \\ D &= 2 \end{aligned}$$

$$\begin{aligned} C &= 12 \\ D &= 2 \end{aligned}$$

$$\begin{aligned} D &= 2 \\ C &= 12 \end{aligned}$$

$$3) \quad \begin{aligned} .05N + .1D &= 40 \\ .1D + .25Q &= 55 \\ .05N + .25Q &= 35 \end{aligned}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 5 & 10 & 0 \\ 0 & 1 & 0 & 0 & 10 & 25 \\ 0 & 0 & 1 & 5 & 0 & 25 \end{array} \right)$$

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$$\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 10 & 25 \\ 0 & 0 & 1 & 5 & 0 & 25 \end{array} \right)$$

Vector equations

$$\begin{aligned} D + C &= 14 \\ 20D + 12C &= 184 \end{aligned}$$

$$\begin{pmatrix} 1 & 1 \\ 20 & 12 \end{pmatrix} \begin{pmatrix} D \\ C \end{pmatrix} = \begin{pmatrix} 14 \\ 184 \end{pmatrix}$$

$$RHS = \begin{pmatrix} -12 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 14 \\ 184 \end{pmatrix} = \begin{pmatrix} -168 \\ 184 \end{pmatrix}$$

$$\begin{aligned} LHS &= \begin{pmatrix} -12 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 20 & 12 \end{pmatrix} \begin{pmatrix} D \\ C \end{pmatrix} \\ &= \begin{pmatrix} -12 & -12 \\ 20 & 12 \end{pmatrix} \begin{pmatrix} D \\ C \end{pmatrix} \end{aligned}$$

$$\begin{pmatrix} 1 & 1 \\ 20 & 12 \end{pmatrix} \begin{pmatrix} D \\ C \end{pmatrix} = \begin{pmatrix} 14 \\ 184 \end{pmatrix}$$

$$\begin{pmatrix} -12 & -12 \\ 8 & 0 \end{pmatrix} \begin{pmatrix} D \\ C \end{pmatrix} = \begin{pmatrix} -168 \\ 16 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 20 & 12 \end{pmatrix} \begin{pmatrix} D \\ C \end{pmatrix} = \begin{pmatrix} 14 \\ 184 \end{pmatrix} \quad \text{multiplied both sides by} \begin{pmatrix} -12 & 0 \\ -12 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -12 & -12 \\ 8 & 0 \end{pmatrix} \begin{pmatrix} D \\ C \end{pmatrix} = \begin{pmatrix} -168 \\ 16 \end{pmatrix} \quad \text{multiply by} \begin{pmatrix} 1 & 0 \\ 0 & 1/8 \end{pmatrix}$$

$$\begin{pmatrix} -12 & -12 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} D \\ C \end{pmatrix} = \begin{pmatrix} -168 \\ 2 \end{pmatrix} \quad \text{multiply} \begin{pmatrix} 1 & 12 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -12 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} D \\ C \end{pmatrix} = \begin{pmatrix} -194 \\ 2 \end{pmatrix} \quad \text{multiply} \begin{pmatrix} -1/12 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} D \\ C \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad \text{multiply} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} D \\ C \end{pmatrix} = \begin{pmatrix} 2 \\ 12 \end{pmatrix}$$

$$\begin{pmatrix} D \\ C \end{pmatrix} = \begin{pmatrix} 2 \\ 12 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 20 & 12 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1/12 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 12 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 12 & 0 \\ -2 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 20 & 12 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 20 & 12 \end{pmatrix} \begin{pmatrix} D \\ C \end{pmatrix} = \begin{pmatrix} 14 \\ 184 \end{pmatrix}$$

matrix $\rightarrow A X = C \rightarrow$ vectors

Solve by multiplying both sides by A^{-1} :

$$\begin{pmatrix} 1 & 0 \\ -20 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 20 & 12 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & -8 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1/8 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & -8 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{8} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -20 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$