Systems of linear equations → Matrices as a simplified notation

$2x+3y=1$
$x-y=2$

$$\begin{pmatrix} 2 & 3 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\left(\begin{array}{cc|c} 2 & 3 & 1 \\ 1 & -1 & 2 \end{array}\right)$$

→ abstraction into "matrix equations"

$$A\vec{x} = \vec{c}$$

→ matrices as a system of "numbers" in their own right

The system of matrices

Think of matrices as a system of things you can add and multiply.

List similarities and differences

| Numbers | Matrices |
|---|---|
| mult. inverses | not always diff. |
| additive inverses | ✓ |
| associative law for mult + add | ✓ ✓ |
| commutativity of multiplication | × |
| identities | ✓ |
| distributive law | ✓ |

For listserv

Why is multiplication of (square) matrices associative?

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} e & f \\ g & h \end{pmatrix}\begin{pmatrix} i & j \\ k & l \end{pmatrix}$$

Matrices as transformations

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$

Use $A$ to define a function

$$\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\mathbb{R}^2 \longrightarrow \mathbb{R}^2$$

$$\begin{pmatrix} x \\ y \end{pmatrix} \longmapsto A\begin{pmatrix} x \\ y \end{pmatrix}$$