

The U.S. Common Core State Standards in Mathematics

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50 US States



16,000 school districts

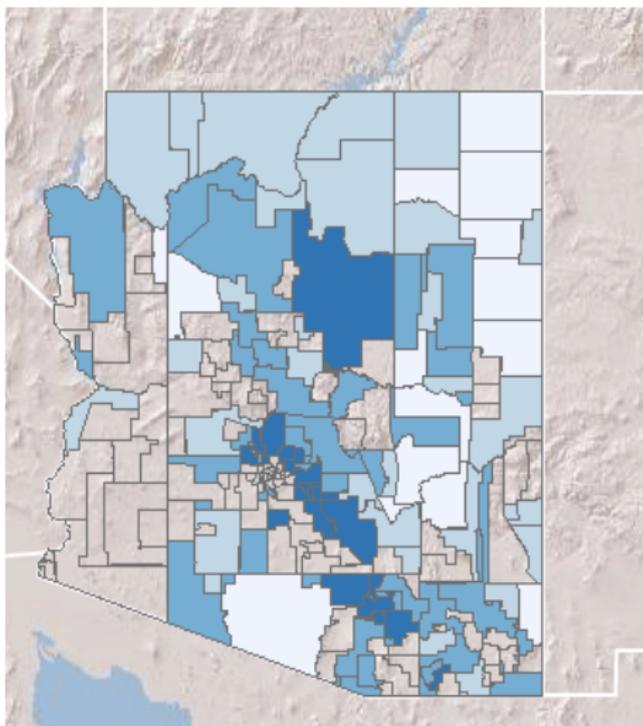


Figure: Arizona school districts

Grade level where different states introduce addition and subtraction of fractions (CSMC)

Grade	Addition & Subtraction of Fractions
1 st grade	2 states
2 nd grade	
3 rd grade	7 states
4 th grade	22 states
5 th grade	9 states
6 th grade	1 state
7 th grade	1 state
8 th grade	
Not specified	

Composite of U.S. state standards, 2005

Note that topics are introduced and sustained in a way that produces no visible structure.

TOPIC	GRADE:	1	2	3	4	5	6	7	8
Whole Number Meaning		■	■	■	■	■	□		
Whole Number Operations		■	■	■	■	■	□		
Measurement Units		■	■	■	■	■	■	■	■
Common Fractions		□	□	■	■	■	■	□	□
Equations & Formulas		□	□	■	■	■	■	■	■
Data Representation & Analysis		■	■	■	■	■	■	■	■
2-D Geometry: Basics		■	■	■	■	■	■	■	■
Polygons & Circles		■	■	■	■	■	■	■	■
Perimeter, Area & Volume			□	□	□	■	■	■	■
Rounding & Significant Figures									
Estimating Computations		□	□	■	■	■	■	■	■
Properties of Whole Number Operations		□	□	□	□				
Estimating Quantity & Size				□					
Decimal Fractions				□	■	■	■	□	□
Relationship of Common & Decimal Fractions					□	□	□		
Properties of Common & Decimal Fractions									
Percentages						□	■	■	□
Proportionality Concepts							■	□	
Proportionality Problems							■	■	■
2-D Coordinate Geometry				□	■	□	□	□	■
Geometry: Transformations		■	■	■	■	■	■	■	■
Negative Numbers, Integers & Their Properties							□	■	□
Number Theory						■	□	□	□
Exponents, Roots & Radicals							□	□	■
Exponents & Orders of Magnitude							□	□	□
Measurement Estimation & Errors		□	□	■	□	■	■	■	□
Constructions w/ Straightedge & Compass									
3-D Geometry		■	■	■	■	■	■	■	■
Congruence & Similarity						□	■	■	□
Rational Numbers & Their Properties							■	■	□
Patterns, Relations & Functions		■	■	■	■	■	■	■	■
Slope & Trigonometry									
Number of topics covered by at least 67% of the states		14	15	18	18	20	25	23	22
Number of additional topics intended by states to complete a typical curriculum at each grade level		8	8	7	8	8	5	6	6

□ - intended by 67% of the states

■ - intended by 83% of the states

■ - intended by 100% of the states



- Mathematics topics intended at each grade by at least two thirds of 21 U.S. States.
- On average a state would have 6–8 more topics per grade level in its complete curriculum.
- From Schmidt, Houang, and Cogan, *American Educator*, 2005.

Composite of high achieving countries

Note that topics are introduced and sustained in a coherent fashion, producing a clear upper-triangular structure.

TOPIC	GRADE:	1	2	3	4	5	6	7	8
Whole Number Meaning		■	■	■	■	■			
Whole Number Operations		■	■	■	■	■			
Measurement Units		□	■	■	■	■	■	■	
Common Fractions				□	■	■	■		
Equations & Formulas				□	■	■	■	■	■
Data Representation & Analysis				□	□	■	■		□
2-D Geometry: Basics				□	■	■	■	■	■
Polygons & Circles					■	■	■	■	■
Perimeter, Area & Volume					■	■	■	■	□
Rounding & Significant Figures					■	■			
Estimating Computations					■	■	■		
Properties of Whole Number Operations				□	■				
Estimating Quantity & Size				□	□				
Decimal Fractions					■	■	■		
Relationship of Common & Decimal Fractions					■	■	■		
Properties of Common & Decimal Fractions						■	■		
Percentages						■	■		
Proportionality Concepts						■	■	■	□
Proportionality Problems						■	■	■	■
2-D Coordinate Geometry					□	□	■	■	
Geometry: Transformations						■	■	■	■
Negative Numbers, Integers & Their Properties							□	■	
Number Theory								■	□
Exponents, Roots & Radicals								■	■
Exponents & Orders of Magnitude								□	□
Measurement Estimation & Errors								□	
Constructions w/ Straightedge & Compass								■	□
3-D Geometry								■	■
Congruence & Similarity									■
Rational Numbers & Their Properties									□
Patterns, Relations & Functions									□
Slope & Trigonometry									□
Number of topics covered by at least 67% of the A+ countries		3	3	7	15	20	17	16	18
Number of additional topics intended by A+ countries to complete a typical curriculum at each grade level		2	6	5	1	1	3	6	3

□ - intended by 67% of the A+ countries ■ - intended by 83% of the A+ countries ■ - intended by 100% of the A+ countries

- A+ countries from 1995 TIMSS
- Topics intended at each grade by at least two thirds of A+ countries.
- On average an A+ country would have 1–6 more topics per grade level in its complete curriculum.

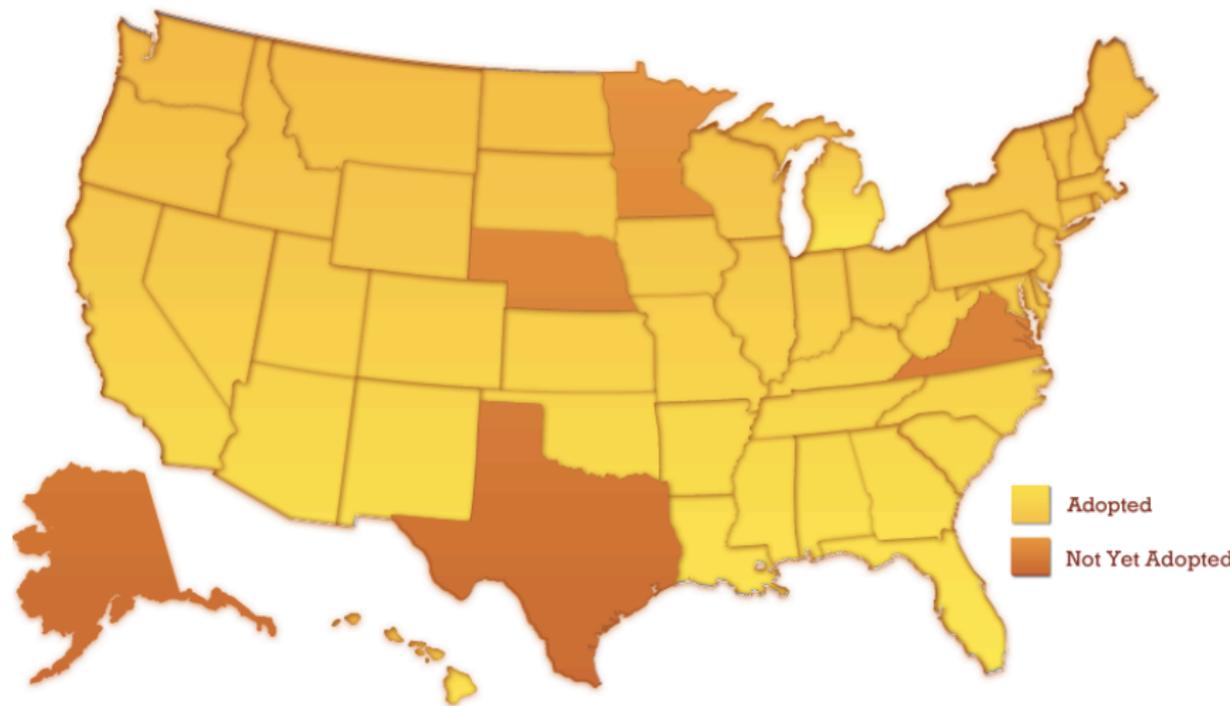
COMMON CORE STATE STANDARDS FOR

Mathematics



- Led by two organizations of states, the National Governors Association (NGA) and the Council of Chief State School Officers (CCSSO)—not directed by the federal government.
- 48 states participated in the development of the standards
- 3-person writing team, 59-person work team, 19-person feedback group
- Based on progressions provided by the work team
- Rapid release and review by work team, feedback group, states, national organizations
- Finished Summer 2010, validated by a validation committee, states started making decisions whether or not to adopt the standards

Adoption of the Common Core State Standards, 2012



Sources considered

- Standards of other countries and states
- Mathematics education research
- The structure of the subject
- National reports
- Commissioned research reviews and progressions
- Opinions of many groups and people

What should standards look like?

O Level Mathematics Syllabus

Secondary One

Secondary One

Topic/Sub-topics	Content
Algebraic representation and formulae	<p>Include:</p> <ul style="list-style-type: none">• using letters to represent numbers• interpreting notations:<ul style="list-style-type: none">* ab as $a \times b$* $\frac{a}{b}$ as $a \div b$* a^2 as $a \times a$, a^3 as $a \times a \times a$, a^2b as $a \times a \times b$, ...* $3y$ as $y + y + y$ or $3 \times y$* $\frac{3 \pm y}{5}$ as $(3 \pm y) \div 5$ or $\frac{1}{5} \times (3 \pm y)$• evaluation of algebraic expressions and formulae• translation of simple real-world situations into algebraic expressions• recognising and representing number patterns (including finding an algebraic expression for the nth term)
Algebraic manipulation	Include:

Number and Operations

STANDARD

for Grades

6–8

*Instructional programs from
prekindergarten through grade 12
should enable all students to—*

Expectations

In grades 6–8 all students should—

Understand numbers, ways of representing numbers, relationships among numbers, and number systems

- work flexibly with fractions, decimals, and percents to solve problems;
- compare and order fractions, decimals, and percents efficiently and find their approximate locations on a number line;
- develop meaning for percents greater than 100 and less than 1;
- understand and use ratios and proportions to represent quantitative relationships;
- develop an understanding of large numbers and recognize and appropriately use exponential, scientific, and calculator notation;

MATHEMATICS STANDARD ARTICULATED BY GRADE LEVEL

GRADE 6

Strand 1: Number Sense and Operations

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Number Sense

Understand and apply numbers, ways of representing numbers, the relationships among numbers and different number systems.

PO 1. Express fractions as ratios, comparing two whole numbers (e.g., $\frac{3}{4}$ is equivalent to 3:4 and 3 to 4).

PO 2. Compare two proper fractions, improper fractions, or mixed numbers.

PO 3. Order three or more proper fractions, improper fractions, or mixed numbers.

PO 4. Determine the equivalency between and among fractions, decimals, and percents in contextual situations.

- NCTM: 14 standards + 7 pages of narrative for Grades 6–8
Number and Operations
- State: 82 standards + 0 pages of narrative for Grade 6
Number and Operations

What should standards look like?

- Flat lists of performance objectives of even grain size, designed to be delivered into the hands of assessment writers without further human intervention?
- Guides to curriculum? A list of things that teachers have to “cover”?
- A statement of what we want our children to understand and be able to do?

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History of testing in the U.S. (courtesy of Cathy Kessel)



The pupil becomes an individual, *American School Board Journal* (1922). (Reprinted by permission of the Department of Special Collections and University Archives, Stanford University Libraries.)

Design principles for the Common Core

Focus attending to fewer topics in greater depth at any given grade level, giving teachers and students time to complete that grade's learning.

Coherence attending to the structure of mathematics and the natural pathways through that structure, where “natural” means taking into account both the imperatives of logic and the imperatives of cognitive development in designing the sequence of ideas.

Rigor balancing conceptual understanding, procedural fluency, and meaningful applications of mathematics.

Standards for Mathematical Content, K–8

K

1

2

3

4

5

6

7

8

Geometry

Measurement and Data

Statistics and Probability

Number and Operations in Base Ten

The Number System

Operations and Algebraic Thinking

Expressions and Equations

Counting and
Cardinality

Number and Operations—
Fractions

Ratios and Proportional
Relationships

Functions

Standards for Mathematical Content, High School

- Number and Quantity
 - ...
- Algebra
 - Seeing Structure in Expressions
 - Arithmetic with Polynomials and Rational Expressions
 - Creating Equations
 - Reasoning with Equations and Inequalities
- Functions
 - Interpreting Functions
 - Building Functions
 - Linear, Quadratic, and Exponential Models
 - Trigonometric Functions
- Modeling
 - ...
- Geometry
 - ...
- Statistics and Probability
 - ...

Standards for Mathematical Practice

- 1 Make sense of problems and persevere in solving them
- 2 Reason abstractly and quantitatively
- 3 Construct viable arguments and critique the reasoning of others
- 4 Model with mathematics
- 5 Use appropriate tools strategically
- 6 Attend to precision
- 7 Look for and make use of structure
- 8 Look for and express regularity in repeated reasoning

Taking focus seriously

K	1	2	3	4	5	6	7	8
Geometry								
Measurement and Data					Statistics and Probability			
Number and Operations in Base Ten					The Number System			
Operations and Algebraic Thinking					Expressions and Equations			
Counting and Cardinality			Number and Operations— Fractions		Ratios and Proportional Relationships		Functions	

Standards not in the number operations domains often support that work:

2.MD.10

Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems.

Taking focus seriously: the case of patterns

- Trickle of pattern standards in elementary school
 - **3.OA.9.** Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.
 - **4.OA.5.** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.
 - **5.OA.3.** Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.
- In middle school, further preparation for functions is provided in the domains Ratios and Proportional Relationships and Expressions and Equations.
- The function concept finally makes its appearance in its own domain in Grade 8, and becomes a major conceptual category in high school.

The payoff for focus

- Taking focus seriously means delaying favored topics until their time, which will be a difficult shift for the educational system in the U.S.
- In high school, the subject matter focus broadens: algebra, functions, geometry, statistics, probability, modeling.
- Focus in high school means not so much a small number of topics as a concentration of skills and practice into a small number of underlying principles.

Coherence

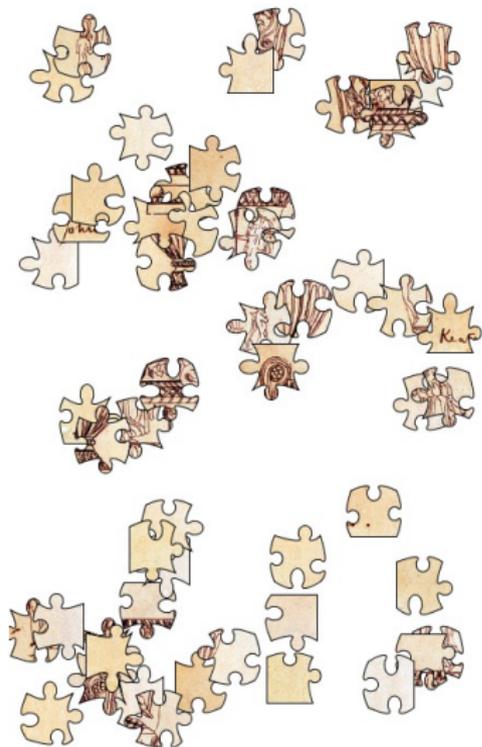


A Grecian Urn

Coherence



A Grecian Urn



Standards for a Grecian Urn



Breaking with the tradition of “equal grain size”

Understand place value (Grade 2)

2.NBT.1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

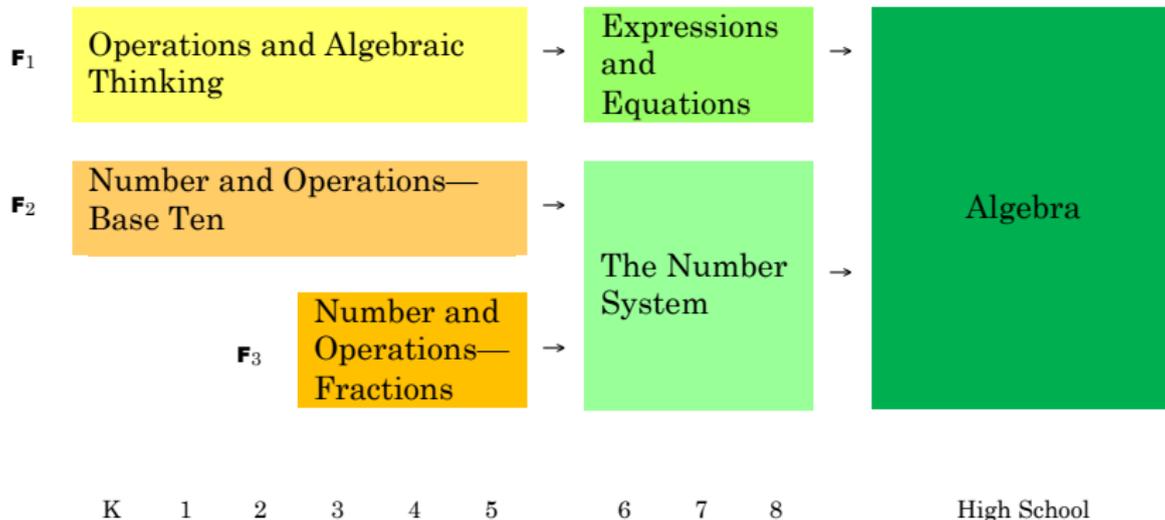
- a 100 can be thought of as a bundle of ten tens—called a “hundred.”
- b The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

2.NBT.2. Count within 1000; skip-count by 5s, 10s, and 100s.

2.NBT.3 ...

2.NBT.4 ...

Coherence flows from focus



“Understand”

- Understand and apply properties of operations and the relationship between addition and subtraction (Grade 2)
- Understand concepts of area and relate area to multiplication and to addition (Grade 3)
- Understand ratio concepts and use ratio reasoning to solve problems (Grade 6)
- Understand congruence and similarity using physical models, transparencies, or geometry software (Grade 8)
- Understand solving equations as a process of reasoning and explain the reasoning (High School)
- Understand and evaluate random processes underlying statistical experiments (High School)

Understanding in Korea

<1st Grade>

A. Numbers and Operations

(a) Numbers up to 100

- ① Understand the concept of 0 and numbers from 1 to 100, and count, write, and read them.
- ② Understand the number system within 100, and compare size.
- ③ Understand the basic concept of the positional numeration system up to 100.
- ④ Break down a number below 10 into two numbers and add the two.
- ⑤ When counting numbers is required, be able to count in groups.

(b) Addition and Subtraction of Simple Numbers

- ① Understand the situations in which addition and subtraction are carried out, and know the meaning of addition and subtraction.
- ② Add and subtract one-digit numbers.
- ③ Using addition expressions where the sum is 10, and the subtraction expression '10-(one-digit number)', find the numbers that add up to 10.

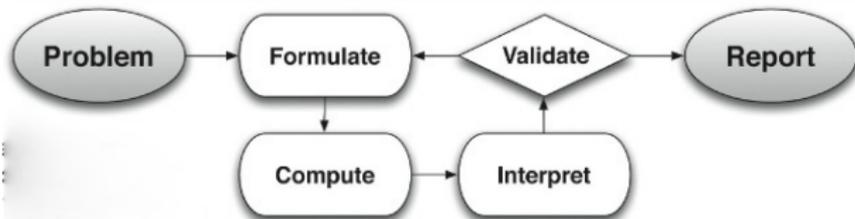
Fluency requirements

- Single digit addition facts in Grade 2
- Single digit multiplication facts in Grade 3
- Addition and subtraction within 1000 in Grade 3
- Addition and subtraction using standard algorithm in Grade 4 (addition and subtraction starts in Grade 1)
- Multiplication using standard algorithm in Grade 5 (multiplication and division starts in Grade 3)
- Division using standard algorithm in Grade 6
- All operations on decimals using standard algorithm in Grade 6
- Solving linear equations with rational coefficients in Grade 7

Putting mathematics to work

7.EE.3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

SMP.4. Model with mathematics.



- Progressions
(<http://ime.math.arizona.edu/progressions/>)
- Illustrative Mathematics Project
(<http://illustrativemathematics.org>).
- My blog, Tools for the Common Core,
<http://commoncoretools.wordpress.com>
- Institute for Mathematics & Education
(<http://ime.math.arizona.edu/commoncore/>)

Comparison of CCSS with A+ composite

Topic	1	2	3	4	5	6	7	8
Whole Number: Meaning	●	●						
Whole Number: Operations	●	●	●	●	●			
Measurement Units	●	●	●	●	●	●	●	
Common Fractions	●	●	●	●	●			
Equations & Formulas			●	●	●	●	●	●
Data Representation & Analysis	●	●	●	●	●	●	●	●
2-D Geometry: Basics			●	●	●	●	●	●
2-D Geometry: Polygons & Circles	●	●	●	●	●	●	●	●
Measurement: Perimeter, Area & Volume		●	●	●	●	●	●	●
Rounding & Significant Figures			●	●	●	●		
Estimating Computations			●	●	●	●		●
Whole Numbers: Properties of Operations	●	●	●	●	●			
Estimating Quantity & Size			●	●	●	●		
Decimal Fractions			●	●	●	●		
Relation of Common & Decimal Fractions			●	●	●	●		
Properties of Common & Decimal Fractions			●	●	●	●		
Percentages					●		●	
Proportionality Concepts					●	●	●	●
Proportionality Problems					●	●	●	●
2-D Geometry: Coordinate Geometry					●	●	●	●
Geometry: Transformations				●				●
Negative Numbers, Integers, & Their Properties					●	●	●	
Number Theory				●			●	●
Exponents, Roots & Radicals						●	●	●
Exponents & Orders of Magnitude							●	●
Measurement: Estimation & Errors							●	●
Constructions Using Straightedge & Compass							●	●
3-D Geometry	●	●			●	●	●	●
Geometry: Congruence & Similarity							●	●
Rational Numbers & Their Properties					●	●	●	●
Patterns, Relations & Functions							●	●
Proportionality: Slope & Trigonometry							●	●

Topic Intended in Common Core Standards ●
 Intended in More Than Half of Top Achieving Countries ■

- The number of extra topics per grade level in CCSS is comparable with A+ countries.