Common Core State Standards for Mathematics

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Composite of high achieving countries

Note that topics are introduced and sustained in a coherent fashion, producing a clear upper-triangular structure.

TOPIC GRADE:	1	2	3	4	5	6	7	8
Whole Number Meaning	■	■	■	▨	▨			
Whole Number Operations	■	■	■	■	▨			
Measurement Units	□	■	■	■	■	■	▨	
Common Fractions			□	■	■	▨		
Equations & Formulas			□	▨	▨	▨	■	■
Data Representation & Analysis			□	□	▨	▨		□
2-D Geometry: Basics			□	▨	▨	▨	■	■
Polygons & Circles				▨	▨	▨	■	■
Perimeter, Area & Volume				▨	▨	▨	▨	□
Rounding & Significant Figures				▨	▨			
Estimating Computations				▨	▨	▨		
Properties of Whole Number Operations				□	▨			
Estimating Quantity & Size				□	□			
Decimal Fractions				▨	■	▨		
Relationship of Common & Decimal Fractions				▨	■	▨		
Properties of Common & Decimal Fractions					▨	▨		
Percentages					▨	▨		
Proportionality Concepts					▨	▨	▨	□
Proportionality Problems					▨	▨	■	■
2-D Coordinate Geometry					□	□	▨	▨
Geometry: Transformations						▨	▨	▨
Negative Numbers, Integers & Their Properties						□	▨	
Number Theory							▨	□
Exponents, Roots & Radicals							▨	▨
Exponents & Orders of Magnitude							□	□
Measurement Estimation & Errors							□	
Constructions w/ Straightedge & Compass							■	□
3-D Geometry							▨	■
Congruence & Similarity								■
Rational Numbers & Their Properties								□
Patterns, Relations & Functions								□
Slope & Trigonometry								□
Number of topics covered by at least 67% of the A+ countries	3	3	7	15	20	17	16	18
Number of additional topics intended by A+ countries to complete a typical curriculum at each grade level	2	6	5	1	1	3	6	3

□ – intended by 67% of the A+ countries ▨ – intended by 83% of the A+ countries ■ – intended by 100% of the A+ countries

- Mathematics topics intended at each grade by at least two thirds of A+ countries.
- A+ countries determined by looking at statistically significant differences in 8th grade scores on 1995 TIMMS
- On average an A+ country would have 1–6 more topics per grade level in its complete curriculum.

Composite of U.S. State Curricula

Note that topics are introduced and sustained in a way that produces no visible structure.

TOPIC GRADE:	1	2	3	4	5	6	7	8
Whole Number Meaning	■	■	■	■	▒	□		
Whole Number Operations	■	■	■	■	▒	□		
Measurement Units	▒	■	■	▒	■	■	■	▒
Common Fractions	□	▒	▒	▒	■	▒	□	□
Equations & Formulas	□	□	▒	▒	▒	■	■	■
Data Representation & Analysis	■	■	■	■	■	■	■	▒
2-D Geometry: Basics	▒	▒	▒	▒	▒	▒	▒	▒
Polygons & Circles	■	■	■	■	■	■	■	▒
Perimeter, Area & Volume		□	□	□	▒	■	■	▒
Rounding & Significant Figures								
Estimating Computations	□	□	▒	▒	▒	▒	▒	▒
Properties of Whole Number Operations	□	□	□	□				
Estimating Quantity & Size			□					
Decimal Fractions			□	▒	▒	▒	□	□
Relationship of Common & Decimal Fractions				□	□	□		
Properties of Common & Decimal Fractions								
Percentages					□	▒	▒	□
Proportionality Concepts						▒	□	
Proportionality Problems						▒	▒	▒
2-D Coordinate Geometry			□	▒	□	□	□	▒
Geometry: Transformations	▒	▒	▒	▒	▒	▒	▒	▒
Negative Numbers, Integers & Their Properties						□	▒	□
Number Theory					▒	□	□	□
Exponents, Roots & Radicals						□	□	■
Exponents & Orders of Magnitude							□	□
Measurement Estimation & Errors	□	□	▒	□	▒	▒	▒	□
Constructions w/ Straightedge & Compass								
3-D Geometry	■	■	■	▒	■	▒	■	▒
Congruence & Similarity					□	▒	▒	□
Rational Numbers & Their Properties						▒	▒	□
Patterns, Relations & Functions	▒	■	■	■	▒	■	■	■
Slope & Trigonometry								
Number of topics covered by at least 67% of the states	14	15	18	18	20	25	23	22
Number of additional topics intended by states to complete a typical curriculum at each grade level	8	8	7	8	8	5	6	6

□ – intended by 67% of the states ▒ – intended by 83% of the states ■ – intended by 100% of the states

- Mathematics topics intended at each grade by at least two thirds of 21 U.S. States.
- On average a state would have 6–8 more topics per grade level in its complete curriculum.
- From Schmidt, Houang, and Cogan, *American Educator*, 2005.

Focus

The composite standards [of Hong Kong, Korea and Singapore] have a number of features that can inform an international benchmarking process for the development of K–6 mathematics standards in the US. First, the composite standards concentrate the early learning of mathematics on the number, measurement, and geometry strands with less emphasis on data analysis and little exposure to algebra. The Hong Kong standards for grades 1–3 devote approximately half the targeted time to numbers and almost all the time remaining to geometry and measurement.

—Ginsburg, Leinwand and Decker, 2009

Mathematics experiences in early childhood settings should concentrate on (1) number (which includes whole number, operations, and relations) and (2) geometry, spatial relations, and measurement, with more mathematics learning time devoted to number than to other topics.

— National Research Council, 2009

Coherence

Schmidt, Houang, and Cogan (2002) say that content standards and curricula are coherent if they are:

> *articulated over time as a sequence of topics and performances that are logical and reflect, where appropriate, the sequential or hierarchical nature of the disciplinary content from which the subject matter derives. That is, what and how students are taught should reflect not only the topics that fall within a certain academic discipline, but* **also the key ideas** *that determine how knowledge is organized and generated within that discipline. This implies that to be coherent, a set of content standards must evolve from particulars (e.g., the meaning and operations of whole numbers, including simple math facts and routine computational procedures associated with whole numbers and fractions) to deeper structures inherent in the discipline. This deeper structure then serves as a means for connecting the particulars (such as an understanding of the rational number system and its properties).*

Concepts

The mathematical process goals should be integrated in these content areas. Children should understand the concepts and learn the skills exemplified in the teaching-learning paths described in this report.

— *National Research Council, 2009*

Because the mathematics concepts in these textbooks are often weak, the presentation becomes more mechanical than is ideal. We looked at both traditional and non-traditional textbooks used in the US and found this conceptual weakness in both.

—*Ginsburg et al., 2005*

There are many ways to organize curricula. The challenge, now rarely met, is to avoid those that distort mathematics and turn off students.

—*Steen, 2007*

Standards for Mathematical Practice

- Make sense of problems and perservere in solving them
- Reason abstractly and quantitatively
- Construct viable arguments and critique the reasoning of others
- Model with mathematics
- Use appropriate tools strategically
- Attend to precision
- Look for and make use of structure
- Look for and express regularity in repeated reasoning

Reason abstractly and quantitatively

Mathematically proficient students make sense of the quantities and their relationships in problem situations. Students bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Grade 3 Fractions

Number—Fractions

3-NF

Fractions as representations of numbers

1. Understand that a unit fraction corresponds to a point on a number line. *For example, 1/3 represents the point obtained by decomposing the interval from 0 to 1 into three equal parts and taking the right-hand endpoint of the first part. In Grade 3, all number lines begin with zero.*
2. Understand that fractions are built from unit fractions. *For example, 5/4 represents the point on a number line obtained by marking off five lengths of ¼ to the right of 0.*
3. Understand that two fractions are equivalent (represent the same number) when both fractions correspond to the same point on a number line. Recognize and generate equivalent fractions with denominators 2, 3, 4, and 6 (e.g., $^1/_2 = {}^2/_4$, $^4/_6 = {}^2/_3$), and explain the reasoning.
4. Understand that whole numbers can be expressed as fractions. *Three important cases are illustrated by the examples* $1 = 4/4$, $6 = 6/1$, *and* $7 = (4 \times 7)/4$. *Expressing whole numbers as fractions can be useful for solving problems or making calculations.*

Fractional quantities

5. Understand that fractions apply to situations where a whole is decomposed into equal parts; use fractions to describe parts of wholes. *For example, to show 1/3 of a length, decompose the length into 3 equal parts and show one of the parts.*
6. Compare and order fractional quantities with equal numerators or equal denominators, using the fractions themselves, tape diagrams, number line representations, and area models. Use > and < symbols to record the results of comparisons.

Fractions as representations of numbers

1. Understand that a unit fraction corresponds to a point on a number line. *For example,* $\frac{1}{3}$ *represents the point obtained by decomposing the interval from 0 to 1 into three equal parts and taking the right-hand endpoint of the first part. In Grade 3, all number lines begin with zero.*
2. Understand that fractions are built from unit fractions. *For example,* $\frac{5}{4}$ *represents the point on a number line obtained by marking off five lengths of* $\frac{1}{4}$ *to the right of 0.*
3. Understand that two fractions are equivalent (represent the same number) when both fractions correspond to the same point on a number line. Recognize and generate equivalent fractions with denominators 2, 3, 4, and 6 (e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$), and explain the reasoning.
4. Understand that whole numbers can be expressed as fractions. *Three important cases are illustrated by the examples* $1 = \frac{4}{4}$, $6 = \frac{6}{1}$, *and* $7 = \frac{4\times 7}{4}$. *Expressing whole numbers as fractions can be useful for solving problems or making calculations.*

Fractional quantities

1. Understand that fractions apply to situations where a whole is decomposed into equal parts; use fractions to describe parts of wholes. *For example, to show $\frac{1}{3}$ of a length, decompose the length into 3 equal parts and show one of the parts.*
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Grade 6 Statistics and Probability

Variability and measures of center

1. Understand that a statistical question is one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question , but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*
2. Understand that a set of data generated by answers to a statistical question typically shows variability—not all of the values are the same—and yet often the values show an overall pattern, often with a tendency to cluster.
   a. A measure of center for a numerical data set summarizes all of its values using a single number. The median is a measure of center in the sense that approximately half the data values are less than the median, while approximately half are greater. The mean is a measure of center in the sense that it is the value that each data point would take on if the total of the data values were redistributed fairly, and in the sense that it is the balance point of a data distribution shown on a dot plot.
   b. A measure of variation for a numerical data set describes how its values vary using a single number. The interquartile range and the mean absolute deviation are both measures of variation.

Summarizing and describing distributions

3. Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
4. Summarize numerical data sets, such as by:
   a. Reporting the number of observations.
   b. Describing the nature of the variable, including how it was measured and its units of measurement. *Data sets can include fractional values at this grade but not negative values.*
   c. Describing center and variation, as well as describing any overall pattern and any striking deviations from the overall pattern.
5. Relate the choice of the median or mean as a measure of center to the shape of the data distribution being described and the context in which it is being used. Do the same for the choice of interquartile range or mean average deviation as a measure of variation. *For example, why are housing prices often summarized by reporting the median selling price, while students' assigned grades are often based on mean homework scores?*

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5. Relate the choice of the median or mean as a measure of center to the shape of the data distribution being described and the context in which it is being used. Do the same for the choice of interquartile range or mean average deviation as a measure of variation. *For example, why are housing prices often summarized by reporting the median selling price, while students assigned grades are often based on mean homework scores?*

High School

- Number and Quantity
- Algebra
  - Seeing Structure in Expressions
  - Arithmetic with Polynomials and Rational Expressions
  - Creating Equations that Describe Numbers or Relationships
  - Reasoning with Equations and Inequalities
- Functions
- Geometry
- Statistics and Probability
- Modeling

Seeing Structure in Expressions

1. Understand that different forms of an expression may reveal different properties of the quantity in question; a purpose in transforming expressions is to find those properties. *Examples: factoring a quadratic expression reveals the zeros of the function it defines, and putting the expression in vertex form reveals its maximum or minimum value; the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*
2. Understand that complicated expressions can be interpreted by viewing one or more of their parts as single entities.
3. Interpret an expression that represents a quantity in terms of the context. Include interpreting parts of an expression, such as terms, factors and coefficients.
4. ...

Comparison of CCSS with A+ composite

Topic	1	2	3	4	5	6	7	8
Whole Number: Meaning	●	●	●	●	●			
Whole Number: Operations	●	●	●	●	●			
Measurement Units	●	●	●	●	●	●	●	
Common Fractions	●	●	●	●	●	●		
Equations & Formulas				●	●	●	●	●
Data Representation & Analysis	●	●	●	●	●	●	●	●
2-D Geometry: Basics		●	●	●	●	●	●	●
2-D Geometry: Polygons & Circles	●	●	●	●	●	●	●	●
Measurement: Perimeter, Area & Volume		●	●	●	●	●	●	●
Rounding & Significant Figures				●	●			
Estimating Computations				●		●		●
Whole Numbers: Properties of Operations	●	●	●	●	●			
Estimating Quantity & Size								
Decimal Fractions				●	●	●		
Relation of Common & Decimal Fractions			●	●	●	●		
Properties of Common & Decimal Fractions					●	●		
Percentages							●	
Proportionality Concepts						●	●	●
Proportionality Problems						●	●	●
2-D Geometry: Coordinate Geometry					●	●	●	●
Geometry: Transformations				●				●
Negative Numbers, Integers, & Their Properties						●	●	
Number Theory				●				
Exponents, Roots & Radicals						●		
Exponents & Orders of Magnitude								
Measurement: Estimation & Errors								
Constructions Using Straightedge & Compass							●	●
3-D Geometry	●	●			●	●	●	●
Geometry: Congruence & Similarity								●
Rational Numbers & Their Properties						●	●	●
Patterns, Relations & Functions								●
Proportionality: Slope & Trigonometry								●

Topic Intended in Common Core Standards ●

Intended in More Than Half of Top Achieving Countries

- The number of extra topics per grade level in CCSS is comparable with A+ countries.