

# A proposal on the foundational mathematics curriculum

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## 1 Executive summary

The Department of Mathematics at the University of Arizona proposes to lead a wholesale review and restructuring of the entire foundational curriculum in mathematics.

The aim of this review is to improve the Department's ability to serve the needs of our students, partner departments, and the University in general by instituting changes which will lead to increased student learning, retention, and graduation, and to do so in a way which maximizes the effectiveness of the resources at our disposal. The review is to be carried out by a blue-ribbon panel appointed by Provost George Davis consisting of faculty from Mathematics and partner disciplines as well as representatives of the administration, local high schools, community colleges, and industry. It is to be supported by the University, possibly with financial assistance from outside groups interested in undergraduate education. The review will cover every aspect of the Department's lower-division (100- and 200-level) course offerings, taking into account the current structure, the process that produced this structure, and the opportunities and challenges ahead for undergraduate mathematics. Topics to be examined include general education graduation requirements, course sequences for colleges and majors, prerequisites, advising, course content, pedagogy, class size, and class structure. The review will set out a realistic direction for lower-division mathematics offerings and make recommendations on the budgetary and physical resources needed by the Department and the University.

Mathematics plays a unique role in the mission of the University of Arizona and over the years the Mathematics Department has developed many innovative and successful initiatives to meet the needs of our students and partner departments. As these needs evolve, we must adjust, improve, and plan carefully in a way that can only be effective if undertaken on a global, curriculum-wide basis. There are currently many competing demands on our resources and many problems where local action to address an issue might well produce an adverse reaction elsewhere in the program. The essential feature of the proposed review

is its global nature—we propose to optimize our program taking into account the interests of all stakeholders.

A review of the scope we have in mind has never been carried out at the University of Arizona, indeed, it may be unprecedented at a major public research university, but the Department is uniquely positioned to carry out such a project. We have a decades-long and nationally recognized tradition of innovation and leadership at all levels of mathematics education, ranging from outreach to local schools and innovative calculus and business math sequences to a showcase “vertically integrated” graduate program and a research and training center for advanced graduate students and post-docs which has consistently drawn the best talent from around the country. This proposal is a continuation of that tradition. Beyond the benefits to our students, a successful restructuring of the type we envision will reassert Arizona’s leadership in mathematics education and could set the terms of debate in foundational education for years to come.

## **2 The unique role of mathematics**

Mathematics plays a unique role in human culture, discourse, and inquiry. Abstraction, analysis, and critical and logical thinking are essential tools of every informed citizen. Mathematics is therefore central to a University education and the Mathematics Department must play a unique role in the educational mission of the university.

The foundational aspects of the Department’s mission are varied and complex. General education requirements describe lower-division mathematics offerings as foundations courses. For those students who will not proceed beyond general education mathematics requirements, this means that their mathematics coursework must provide them with all the mathematical and analytical knowledge and skills necessary to function in our modern technical society. For many other students, the mathematical foundations they need are acquired in sequences of courses specifically designed to contribute to their future major. It is a huge and hugely complicated responsibility to meet the needs of all of our students and partner departments.

## **3 A tradition of innovation and leadership**

The Department’s tradition of innovation and leadership is particularly strong in undergraduate education. Over the years we have responded to the needs of students and partners with many innovative texts, courses, and course sequences. For example, the Calculus Consortium has written a series of influential and best-selling textbooks for the calculus sequence which have changed forever calculus instruction in the US. Four of the principal members of this group are UA faculty. In 1995, the department won the prestigious Hesburgh award, with a congratulatory letter from President Clinton, for faculty development to en-

hance undergraduate teaching. More recently, a pioneering technology-based mathematics sequence for business majors has been developed in a partnership between the Mathematics Department and the Eller School of Management. The resulting electronic textbook, whose author won the ICTCM Award for Excellence and Innovation, has been published by the Mathematical Association of America and is being widely adopted. The American Mathematical Society profiled our department as an example of best practices in the volume *Towards Excellence*. Many other efforts in curriculum development, use of technology, and faculty and GTA training and evaluation have been carried out in recent years.

There have also been regular efforts to review, adjust, and improve our offerings as circumstances change. Specific examples include a review of freshman mathematics in 1983 (whose reforms were documented in a report by the Conference Board on Mathematics Sciences), the introduction of differing strands of mathematics for various colleges and majors in the early 1990s, our innovative Co-op programs with local high schools (started in 1987) and Pima Community College (started in 1995), and efforts to adjust admissions standards in 1988 and 1998. The department has also been active in efforts to reach out to other disciplines. For example, we organized a national workshop on the mathematical needs of business students in 2002 and we are currently organizing a similar meeting on the mathematical education of journalism students. Both of these meetings were part of a larger project on the foundational mathematics curriculum sponsored by the Mathematical Association of America. Two members of the Mathematics faculty are on the advisory board of this project.

## 4 The need for a global review

Although there have been regular improvements and maintenance, neither the University nor the Department has ever conducted a comprehensive study of the lower-division mathematics curriculum taking into account its full range of responsibilities. There are several reasons why it is important to carry out a global review now:

- The University is under severe strain due to increased demands combined with limited, even decreasing, resources, including an insufficient number of faculty, limited space, and tight budgets. The Department will soon face a series of difficult decisions concerning the allocation of its resources. Any change in lower-division course offerings will be felt across campus and must be done in consultation with all of our partner departments.
- These strains are exacerbated by the fact that we are experiencing dramatically increased demand for the Mathematics major and for upper-division Mathematics courses. These important needs can only be met by regular faculty and in small course sections and as a result, there is even greater strain on our limited resources.

- It is crucial that we plan both for the current strained environment and for the future so that we may effectively utilize new resources, including faculty, space and increased funding, if and when they become available.
- Another important reason for conducting a global review now is that there is increasing recognition of the need to collaborate with partner departments at a much deeper level than in the past. The increase in the quantitative skills needed in the life sciences is one of the most visible factors driving this trend.
- Finally and most importantly, we believe that a global review and restructuring will allow us to better meet our responsibilities—by considering our offerings as a whole we hope to develop new initiatives, maximize the effectiveness and impact of previous initiatives, and create synergies between them.

## 5 A global examination of the foundational curriculum in mathematics

The Mathematics Department proposes a review and possible restructuring of the entire foundational mathematics curriculum, ranging from Algebra, Trigonometry, and Math in Modern Society, through the Calculus, Differential Equations and Linear Algebra sequences, to Discrete Mathematics, Statistics, and teacher preparation courses. The goal is to optimize University resources and policies to facilitate student success in mathematics, understood in a broad sense. Specifically, our goals are to

- increase student success in lower-division mathematics, leading to improved student retention and graduation rates
- meet the foundational needs of all our partner disciplines, from the physical sciences, life sciences, and engineering to the social sciences and humanities
- ensure that students have appropriate mathematical education for all segments of the workforce
- increase the opportunities for underrepresented students to graduate and go on to advanced degrees by improving their mathematical education

Every aspect of the foundational curriculum will be considered. Some topics already under discussion include:

- How can the Mathematics program better meet the University's goals for education?
- What mechanisms can be put in place to measure *quantitatively and not just anecdotally* the success or failure of proposed reforms?

- Should entrance requirements be modified? Should students be required to declare a major sooner to facilitate major-specific tracks in basic courses?
- Are our students being appropriately advised, in particular about Mathematics offerings? Are prerequisites being appropriately enforced? How can our freshman placement program be made more effective and efficient?
- What should the role of technology, in particular handheld computers, be in the lower-division curriculum, both in content and in pedagogy?
- How can we evaluate the extent to which alternative learning strategies are appropriate for our students?
- Mathematics courses are currently taught almost exclusively in sections of 30-35 students. What are the relative merits and demerits of smaller sections and of large lectures with discussion sections?
- What is the appropriate role for Graduate Teaching Assistants in foundational mathematics courses?
- How can we meet the large and growing teacher training needs now that the College of Education has turned over this responsibility to the Mathematics Department?
- How can we improve the performance of our students in the most basic mathematics courses such as Algebra, Trigonometry, Statistics, and Calculus?
- Do our current courses in Algebra meet the needs of students for whom they are required, in particular in the biological sciences?
- More generally, is there demand for major-specific tracks of basic courses such as Algebra, Statistics, and Calculus? In addition to its one large, general education algebra course, the Department has two well-defined programs: one for engineering and science students and another for business students. Do biology, agriculture, or social science students need mathematics course sequences designed specifically for their needs? If so, how can these be efficiently offered and managed?
- What array of statistics courses will meet the needs of students, in particular from the life sciences, social sciences, and engineering?
- The main calculus sequence for scientists and engineers begins with a 5-unit course, continues with a 3-unit course, and concludes with a 4-unit course. It is also available as a 3-3-4 track. Is this optimal?
- Does the foundational mathematics curriculum provide appropriate preparation for the most mathematically sophisticated majors, including the mathematics major itself?

No doubt other questions will arise as the project progresses. Everything is on the table subject to the goals above. Increasing student success will be the ultimate standard.

We propose that this global examination be carried out by a blue-ribbon panel appointed by the Provost of the University and including representatives from the following groups:

- faculty from the Department of Mathematics in sufficient numbers to provide assistance in all breakout groups.
- faculty from partner disciplines, including the sciences, engineering, business, nursing and medicine, and the social sciences
- the upper administration of the University of Arizona
- local school districts
- Pima Community College and UA South
- local business and industry
- experts on diversity and underrepresented student issues
- university budget and space experts
- an advisory board of leaders from business, government, and academia

Clearly this is a large undertaking which entails a substantial commitment of resources, including funding, faculty and administration time, and space. Moreover, in order that the review proceed seriously, there must be a strong commitment to implement the recommendations of the review panel, including generating and allocating needed resources.

Among many other positive effects, the successful completion and implementation of such a review will reassert Arizona's national leadership in undergraduate mathematics.

## **6 What this proposal is not**

This proposal is not an offer to make marginal change or patch-work improvements. The required effort and the risks are simply too large. Nothing less than a wholesale examination and reform of the foundational curriculum is being proposed.