Here are some counter-example type problems that you can play with. There may be more in the book. I may or may not extract problems from this pool. The following are false statements. Find a counter-example for each statement:

1. Let $A$ be $m \times n$. If $Av = 0$ for some $v \in \mathbb{R}^n$, then $A = 0$.

2. Let $A$ be $m \times n$ and $B, C$ be $n \times p$. If $AB = AC$, then $B = C$.

3. $A, B$ be $n \times n$. If det$(A) = $ det$(B)$, then $A \sim B$.

4. If $AB = 0$, then $A = 0$ or $B = 0$.

5. If $A, B$ have the same set of eigenvalues, then $A, B$ have the same eigenspaces.

6. If $A, B$ have the same set of eigenvalues, then $A \sim B$.

7. If $A$ has eigenvalue $\lambda$ with eigenvector $u$ and $B$ has eigenvalue $\mu$ with eigenvector $v$, then $A + B$ has eigenvalue $\lambda + \mu$ with eigenvector $u + v$.

8. If $A$ has eigenvalue $\lambda$ and $B$ has eigenvalue $\mu$, then $AB$ has eigenvalue $\lambda \mu$.

9. If $A, B$ are symmetric, then $AB$ is symmetric.

10. $(AB)^{-1} = A^{-1}B^{-1}$.

11. $T \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ x^2 \end{pmatrix}$ is a linear transformation.

12. If $U, W$ are subspaces of $V$. Then $U \cup W$ is a subspace of $V$ also.