## Extra Credit Session 2 – chapter 6

1 Let $V = \mathcal{D}$ is the vector space of differentiable functions and $W = \{f \in \mathcal{D} | f'(x) + f(x) = 0\}$. Show that $W$ is a subspace of $V$.

2 $W = \left\{ A \in M_{22} | A = \begin{pmatrix} x & -x \\ y & z \end{pmatrix} \right\}$ is a subspace of $M_{22}$. Find a basis for $W$.

3 Find the coordinate vector for $p(x) = 2x^2 + x + 3$ in the basis $\mathcal{B} = \{1, x - 1, (x - 1)^2\}$ for $\mathcal{P}_2$.

4 Let $T : P_2 \to P_2$ where $T(1) = x$, $T(x - 1) = x^2 + 1$, $T((x - 1)^2) = 1 - x$. Find $T(2x^2 + x + 3)$.

5 Let $B = \{\mathbf{e}_1, \mathbf{e}_2\}$ and $B' = \{\mathbf{e}_1 + \mathbf{e}_2, 2\mathbf{e}_1 + 3\mathbf{e}_2\}$ are bases for $\mathbb{R}^2$. Find the change of basis matrix from $B$ to $B'$.

6 Let $B$ and $B'$ as defined in problem 5. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T(\mathbf{e}_1) = 3\mathbf{e}_1 - 2\mathbf{e}_2$, $T(\mathbf{e}_2) = \mathbf{e}_1 + 4\mathbf{e}_2$. Write down the matrix representation for $T$ in the standard basis $B$. Then use the change of basis matrix from problem 5 to find the matrix representation for $T$ in basis $B'$.

True/False. Give reasons.

7 Every vector space has a finite basis.

8 If $T : V \to W$ is a function from a vector space $V$ to a vector space $W$, then $T$ is linear if and only if $T(\mathbf{x} + \mathbf{y}) = T(\mathbf{x}) + T(\mathbf{y})$.

9 If $T$ is a linear transformation, then $T$ maps a linearly independent set to a linearly independent set.

10 The empty set is a subspace of every vector space.