Extra Credit Session 3 – chapter 4

1 Let $A = \begin{pmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{pmatrix}$. Use Gaussian elimination to compute its determinant.

2 Let A be as defined above. Use Cramer's Rule to compute the second variable of the system $A \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$.

3 Let A be as defined above. Its characteristic polynomial is $(\lambda - 3)^2(\lambda - 5)$. Find its eigenvalues and a basis for its eigenspaces. Show that A is diagonalizable by finding the invertible matrix P and diagonal matrix D such that $D = P^{-1}AP$.

4 Use the diagonalization found in the previous problem to solve the system of differential equation $\mathbf{x}' = A\mathbf{x}$ where A is as defined above.

5 Let $A = \begin{pmatrix} 3 & 4 & 3 \\ 5 & 7 & 2 \\ 0 & 0 & 1 \end{pmatrix}$. Compute $\operatorname{adj}(A)$ and use this to compute A^{-1}.

6 Find the eigenvalues of the matrix $A = \begin{pmatrix} 11 & -15 \\ 6 & -7 \end{pmatrix}$. If this matrix describes a dynamical system $x_n = Ax_{n-1}$, what type of fixed point is the zero vector?

True/False. Give reasons.

7 If A is an $n \times n$ matrix and $A\mathbf{x} = \mathbf{0}$ for some $\mathbf{x} \neq \mathbf{0}$, then $\det(A) = 0$.

8 If λ is an eigenvalue of A, then the geometric multiplicity of $\lambda = \operatorname{rank}(A - \lambda I)$.